



Mathematical evaluation of forward osmosis efficiency for methylene blue removal using a flat-sheet TFC membrane

Zainab Ali Ibrahim ^{a, b, *}, Ahmed Faiq Al-Alawy ^a, Abdullah Saleh Mohammed Khaled ^c

^a Department of Chemical Engineering, College of Engineering, University of Baghdad, Baghdad, Iraq

^b Northern Technical University, Mosul, Iraq

^c Hydrocarbon and Petrochemical Process Department, Faculty of Engineering, University of Pannonia, Veszprém, Hungary 8200

Abstract

Forward osmosis (FO) has gained a lot of research interest due to its many possible uses in desalination and wastewater reuse. The FO is a cost-effective method because it relies on concentration differences at the membrane surfaces and does not require external hydraulic pressure. This work deals with the design and construction of a forward osmosis system containing a membrane cell in which the thin film composite (TFC) flat sheet membrane is fixed, to carry out practical experiments by using of NaCl as the draw solution (DS), and the methylene blue (MB) dye is used as the feed solution (FS), and then employing the resulting experimental data in building and verifying the mathematical model of the FO process. The concentration of NaCl is from 5 to 70 g/L, and the concentration of MB is 20 ppm with a feeding pressure of 2 bar. A solution diffusion model was developed to predict water flux theoretically and compared with experimental results using different DS concentrations. The effect of concentration polarization was also studied. The results showed that the flux increased from 3.65 to 9.52 LMH, and the osmotic pressure at the bulk of the membrane increased linearly from 4.01 to 56.16 bar when the DS concentration increased from 5 to 70 g/L. Also, the recovery of pure water increased with increased concentration of NaCl. The results showed that there was a good agreement between the experimental and theoretical results obtained from the mathematical model, where the correlation coefficient (R^2) was 98.89%.

Keywords: Forward osmosis; Concentration polarization; Sodium chloride; Mathematical Modeling; Osmotic pressure.

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1- Introduction

Pure water is one of the most important resources on Earth, yet it is inaccessible to billions of people. The scarcity of clean water is worsening due to climate change, overuse, and rapid industrialization. The growing demand from cities, industry, and agriculture increases pressure on limited water supplies [1-3]. Industrial dyes are the major source of increasing environmental contamination, representing the largest category of organic pollutant substances. Commonly utilized industrial dyes include Sulphur dyes, reactive dyes, vat dyes, mordant dyes, acidic dyes, basic dyes, and azo dyes [4]. Removing these dyes is important for ecosystem health because they are used in many industries, such as printing, paper, textiles, food, pharmaceuticals, and rubber [5, 6].

Methylene blue (MB), also known as 3,7-bis(dimethylamino)phenothiazine chloride or tetramethylthionine chloride, is an organic dye commonly used in the dyeing of paper, wool, and silk. In addition, MB is extensively utilized in the manufacturing processes of pharmaceuticals, cosmetics, and food products [7]. Due to its widespread industrial application, substantial quantities of wastewater containing MB are released into

both surface and groundwater. MB exposure can cause ocular burns, which may result in irreversible eye damage in humans and animals. The acceptable concentration of MB differs among regulatory agencies and municipalities, but the Environmental Protection Agency (EPA) guidelines generally suggest a sufficient level of 0.2 mg/L [8, 9].

There are three main approaches to remove dyes from wastewater: biological [10], chemical [11], and physical [12]. Due to factors such as pressure, osmosis, electricity, enhanced membrane performance, and cost-effectiveness, membrane filtration has recently garnered considerable attention as a means of treating dye wastewater. Membrane technology is widely recognized for its distinctive advantages, including mechanical robustness, enhanced resistance to fouling, and high solute selectivity, which together make it an efficient and versatile separation approach for water and wastewater treatment applications. Depending on the driving force and membrane characteristics, membrane processes can be classified into electrically driven and pressure-driven technologies. Electrodialysis (ED) is an electrically driven membrane process that utilizes an applied electric potential to transport ions selectively through ion-



*Corresponding Author: Email: zainabalaskari@ntu.edu.iq

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exchange membranes, making it particularly suitable for desalination and the removal of charged species. In contrast, pressure-driven membrane processes rely on hydraulic pressure as the driving force and employ membranes with progressively decreasing pore sizes to separate contaminants of different sizes. Microfiltration (MF) is used to remove suspended solids, bacteria, and large colloidal particles. Ultrafiltration (UF) operates with smaller pore sizes, enabling the removal of macromolecules, proteins, and viruses. Nanofiltration (NF) provides partial rejection of multivalent ions and small organic molecules, while reverse osmosis (RO) employs the densest membranes, allowing for the removal of monovalent ions and dissolved salts, thus producing high-purity water [13-15].

Forward osmosis, also called direct osmosis, is a membrane process driven by osmotic pressure, utilizing a highly selective membrane, and is an advanced technique for the purification and treatment of various types of wastewaters. FO is a method that could substantially lower the expenses associated with desalinating saline water sources [16]. The FO method uses a semipermeable membrane and two solutions: a feed solution and a draw solution. The osmotic pressures of the solutions differ. The difference in osmotic pressure drives the forward osmosis process. It moves water from the feed solution, with low osmotic pressure, to the draw solution, with higher osmotic pressure, through a semipermeable membrane [17, 18]. The FO has a low risk of membrane fouling, operates at reduced hydraulic pressure, and reduces energy costs [19]. The FO membrane and the properties of the employed draw solution are two critical aspects that govern FO processes, as they collectively determine the viability of this technology [20].

It is important to select a draw agent compatible with the FO membrane and chemically stable [21]. Additionally, FO permeate is not a consumable product; rather, it is a mixture of the draw solution and product water. Clean water is extracted from DS, which is then regenerated. The usage of inappropriate DS and regeneration methods may result in excessive energy consumption during this process. FO may consume merely 20% of the electrical power required by alternative desalination methods [22]. Smart draw solutions are a new category of DS. They encompass various categories, including smart hydrogel polymers, thermo-responsive materials, ionic liquids, magnetic nanoparticles, and traditional monovalent inorganic salts, such as NaCl, which are extensively utilized for extracting significantly large water fluxes. Responsive draw solutes are a category of intelligent draw solutes capable of significantly reducing the osmotic pressure of diluted draw solutions upon exposure to external stimuli. Responsive draw solutes can be extracted from the permeate water through external stimulation, serving as a straightforward initial stage in the regeneration of draw solutes. Nevertheless, an additional separation step may still be required to fully purify the thus-produced water by the use of nanofiltration or reverse osmosis. Therefore, employing suitable responsive draw solutes can

significantly alleviate the challenges associated with the recovery process of non-responsive draw solutes [23, 24].

This research aims to develop a mathematical model of the forward osmosis process based on experimental data extracted from the laboratory system, and to study the extent to which the model matches the experimental results to evaluate its efficiency in predicting flux values.

2- Experimental work

A schematic FO unit was utilized in the experiments and is shown in Fig. 1. The experimental work is discussed in our previously published paper [25].

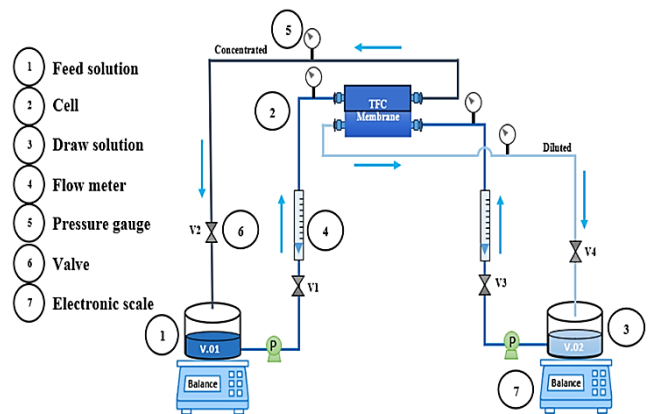


Fig. 1. Schematic of a forward osmosis setup

3- Mathematical modelling

3.1. Osmotic pressure

A solution's osmotic pressure is commonly utilized to denote the concentration of the solution. It is a colligative feature of a material, as it is concentration-dependent and independent of the material's chemical makeup [26]. Osmotic pressure in a dilute solution agrees with the ideal gas law and may be calculated based on the solution's temperature and concentration [27]. A solution's adequate osmotic pressure is the part of its total osmotic pressure that controls the solvent's propensity to cross a boundary, most commonly a semipermeable membrane [28, 29]. The equation of osmotic pressure established by van't Hoff is extensively utilized.

$$\pi = n \Phi C R_g T \quad (1)$$

The variables in this equation are the following: the molar concentration (C), the osmotic coefficient (Φ) for NaCl equal to 0.93, R_g the gas constant ($R_g=0.0821$ L atm /mol °K), and T, the absolute temperature in °K and n, the van't Hoff factor, which describes the number of individual particles of a compound dissolved in a solution, for example, is equal to 2 for sodium chloride and for methylene blue [30].

In the forward osmosis technique, the feed water flows towards the highly concentrated draw solution, leaving solutes behind, due to the difference in osmotic pressure

generated through a membrane that divides the feed and draw solutions. In a forward osmosis system, the water flux (J_w) is defined by [31]:

$$J_w = A (\sigma \Delta \pi - \Delta P) \quad (2)$$

The equation includes the following variables: hydraulic pressure (ΔP), osmotic pressure differential ($\Delta \pi$), reflection coefficient (σ), which can be presumed to be one, water flux (J_w), and the membrane's water permeability constant (A).

The water flux in the forward osmosis system is dependent upon the flux over the membrane's selective layer and is expressed as [32]:

$$J_w = A(\Delta \pi_{eff} - \Delta P) \quad (3)$$

$$J_w = A[(\pi_D - \pi_F) - (P_D - P_F)] \quad (4)$$

Where the difference between π_D and π_F , which is also known as the effective osmotic pressure difference ($\Delta \pi_{eff}$), represents the osmotic pressure through the FO membrane selective layer, and ΔP represents the difference between P_D and P_F , which is also called the applied hydraulic pressure on the DS and FS sides.

The permeability constant can be calculated from the equations of fouling [33]:

$$\mu_w R_m = \frac{\Delta P}{J_w} \quad (5)$$

$$\mu_w R_f = \frac{\Delta P}{J_f} - \mu_w R_m \quad (6)$$

$$R_t = \mu_w R_m + \mu_w R_f \quad (7)$$

$$A = \frac{1}{R_t} \quad (8)$$

Where R_t represents the total of different resistances, R_m the membrane's fresh resistance just before separation, ΔP the hydraulic pressure, J_w the water flux of pure water before separation, μ_w the pure water viscosity, R_f the membrane's fouled resistance, and J_f the water flux of pure water after separation.

3.2. Concentration polarization

Concentration polarization (CP) is a prevalent phenomenon observed in both pressure-driven and osmosis-driven membrane processes; however, it is more pronounced in osmotically driven processes such as FO due to their intrinsic characteristics. This phenomena in the FO process results from the concentration disparities between the feed solution and the draw solution. CP denotes the concentration of FS on the membrane surface, arising from water flux across the membrane while concurrently diluting the DS on the other side. This results in inadequate water flux and reverse salt flux due to considerable mass transfer resistance that arises both inside and surrounding membranes during the osmosis process. Consequently, CP decreases the real effective

osmotic pressure gradient under the osmotic pressure differential between the FS and DS [34].

The existence of two separate solutions on either side of the membrane leads to two different kinds of concentration polarization: concentrative concentration polarization on the membrane surface adjacent to the feed solution and dilutive concentration polarization on the surface of the membrane adjacent to the draw solution. Due to the often-asymmetric construction of dense synthetic polymeric membranes, which have a thin active layer atop a porous support layer, concentration polarization effects may manifest either on the active layer (ECP) or within the support layer (ICP). The CP phenomena of a process operating in FO mode (with the active layer of the membrane facing the FS and the porous support layer of the membrane facing the DS) include dilutive internal concentration polarization (DICP) and concentrative external concentration polarization (CECP). While in PRO mode (with FS facing the membrane porous support layer and DS facing the active layer), include concentrative ICP (CICP) and dilutive ECP (DECP), as shown in Fig. 2 [35].

CECP is a well-established phenomenon in pressure-driven membrane technology. In the FO process, a comparable event occurs on the side of the feed solution, which faces the membrane's active layer. The osmotic pressure of the feed at the membrane's active layer ($\pi_{F,m}$) is expressed as [36]:

$$\pi_{F,m} = \pi_{F,b} \exp\left(\frac{J_w}{k_F}\right) \quad (9)$$

Here, $\pi_{F,b}$ denotes the bulk osmotic pressure on the feed side, where $\pi_{F,m} > \pi_{F,b}$, and k_F denotes the boundary layer's mass transfer coefficient on the feed side. The related relationship expresses the latter:

$$k_F = \frac{Sh D_F}{d_h} \quad (10)$$

In this case, D_F is the feed solution's diffusion coefficient, d_h is the feed channel's hydraulic diameter, and Sh is the Sherwood number.

$$d_h = \frac{2ab}{a+b} \quad (11)$$

Where the cell's dimensions are represented by a and b .

The Sherwood number was defined by the relationship that follows, depending upon the flow conditions within the channel:

$$Sh = 1.85 \left(Re Sc \frac{d_h}{L} \right)^{0.33} \quad (\text{laminar flow}) \quad (12)$$

$$Sh = 0.04 Re^{0.75} Sc^{0.33} \quad (\text{turbulent flow}) \quad (13)$$

Where Re refers to the Reynolds number, Sc to the Schmidt number, and L to the channel length:

$$Sc = \frac{\mu_F}{\rho_F D_F} \quad (14)$$

$$Re = \frac{\rho_F U_F d_h}{\mu_F} \quad (15)$$

Where ρ_F represents the density of the feed solution, μ_F is the dynamic viscosity of the feed solution, and U_F is the velocity of the feed.

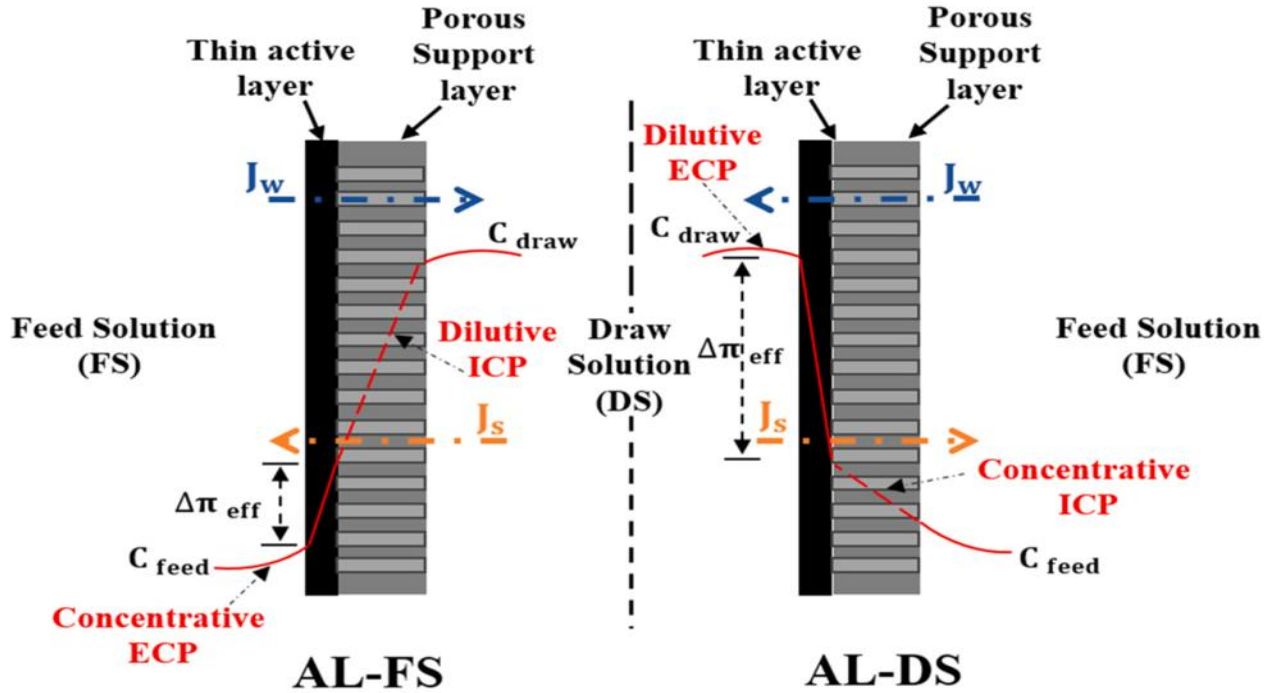


Fig. 2. The FO composite membrane's orientations, AL-FS and AL-DS, exhibit concentration polarization

On the other side of the membrane, within the support layer, is where DICP takes place; this side of the membrane is oriented towards the DS. The draw solutes are displaced and transported away from the interface of the active layer of the membrane by the convective flux from the feed. Due to this, the effective driving force is reduced because there are fewer draw solutes in nearby areas of the active layer. Near the active layer membrane interface, the following equation gives the osmotic pressure ($\pi_{D,i}$) inside the porous support layer [37]:

$$\pi_{D,i} = \pi_{D,b} \exp(-J_w K_D) \quad (16)$$

Here, $\pi_{D,b}$ is the draw solution osmotic pressure at the bulk, where $\pi_{D,i} < \pi_{D,b}$ and K_D represent the solute resistivity.

Within the porous support layer, the solute resistivity for the diffusion of DS, was expressed as:

$$K_D = \frac{t_m \tau}{D_D \varepsilon} = \frac{S}{D_D} \quad (17)$$

To find the value of D_D by using the OLI stream analyser from [36]:

$$D_D = (-1.025 \times 10^{-10})C + 1.518 \times 10^{-9} \quad (18)$$

Where the membrane structural parameter (S), the thickness (tm), tortuosity (τ), (C) the concentration of DS, and porosity (ε) of the support layer, the draw solute types and concentration will likely affect the value of D_D , but the values of t , τ , and ε remain constant for any particular membrane, therefore, K_D is a quantity that reflects the

severity of ICP in the FO process; it measures the solute's effectiveness in diffusing into and out of the support layer. Using the following flux relationship for the FO mode, as reported by [38], to find the value of K_D :

$$K_D = \left(\frac{1}{J_w}\right) \ln \frac{B + A\pi_{D,b}}{B + J_w + A\pi_{F,m}} \quad (19)$$

The membrane's salt permeability coefficient, abbreviated as B, can be calculated from the experiments rejection in the RO system.

$$B = \frac{A(1-R_S)(\Delta P - \Delta \pi)}{R_S} \quad (20)$$

Where R_S refers to the membrane's salt rejection, and ΔP is the pressure applied through the RO rejection experiment.

Consequently, the water flux represented by Eq. 4 must be adjusted to incorporate the CECF and DICP phenomena, resulting in the following modified equation for water flux in the FO system:

$$J_w = A[\pi_{D,i} - \pi_{F,m} + P_F] \quad (21)$$

Where P_D equals zero.

Eqs. 9 and 16 are substituted into Eq. 21 to get:

$$J_w = A[\pi_{D,b} \exp(-J_w K_D) - \pi_{F,b} \exp\left(\frac{J_w}{k_F}\right) + P_F] \quad (22)$$

The parameters in Eq. 22 involve measurable system conditions and membrane characteristics. The characteristics of the flat-sheet membrane, solution

properties, and operating conditions are listed in Table 1. The Goal Seek feature in Excel is used to solve the FO mathematical model to obtain water flux. The FO mathematical model's flow chart is shown in Fig. 3.

Table 1. Parameters employed in FO modeling

Parameters	Values
Membrane type	TFC flat sheet
a	6 cm
b	2 mm
d_h	0.003871 m
Q_D	200 mL/min
Q_F	200 mL/min
A	1.5 L/m ² .h.bar
B_{NaCl}	0.38 LMH
S	0.0003 m
P_F	2 bar
$C_{D,i}$	5, 10, 20, 35, 70 g/L NaCl
C_F	20 ppm MB
T	30°C

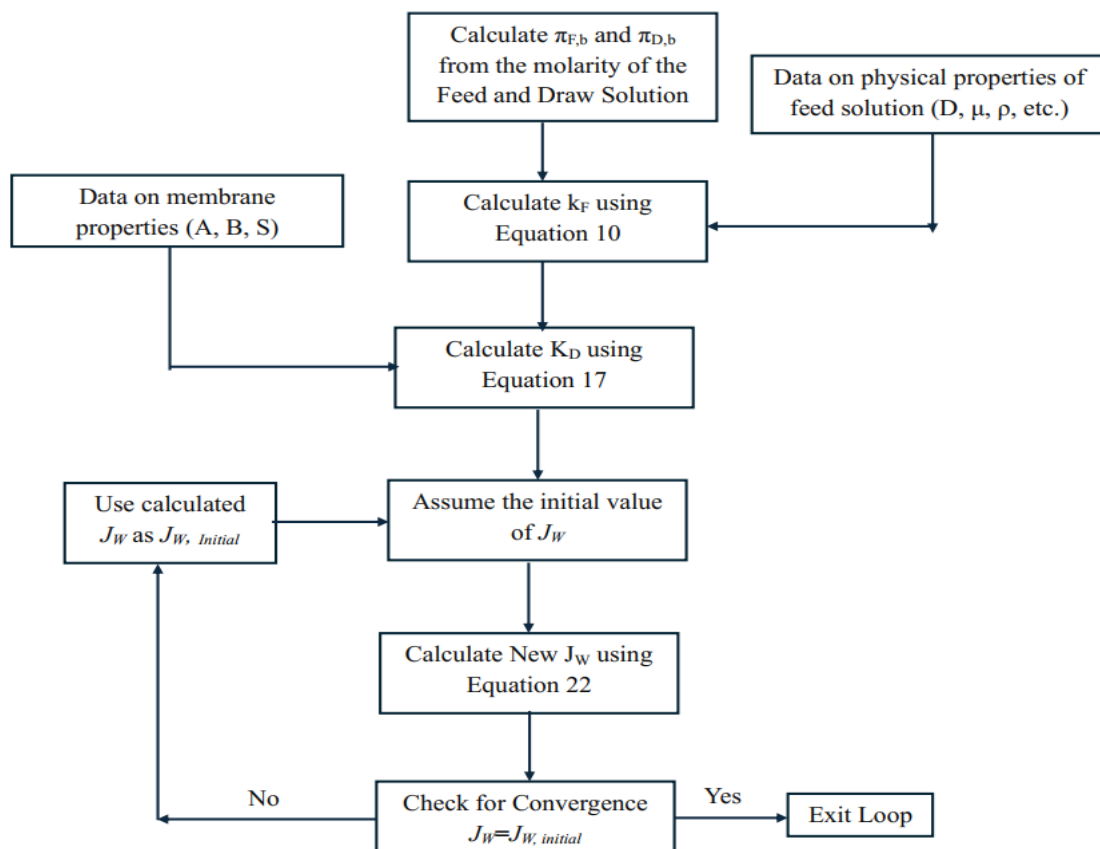


Fig. 3. The FO mathematical model's flow chart

4- Results and discussion

This study theoretically determined the water flux utilizing the diffusion solution model. Numerous draw solution concentrations have been used to compare theoretical and experimental results. The water flux equation is affected by several factors, including feed and draw solution osmotic pressure and concentration, the feed mass transfer coefficient, the draw solute diffusivity in the porous support layer, the water permeability coefficient, and other variables, is constant, such as the pressure drop on the feed side equal 2 bar and the flow rates of FS and DS equal 200 mL/min. The mathematical

model uses Eqs. 1-22 to forecast theoretical water fluxes, incorporating the impacts of concentration polarization.

Eq. 22 depends on many variables, such as membrane permeability, which is one of the most significant factors directly influencing the value of flux, as clearly demonstrated in the mathematical equation for flux. The more permeable the membrane to water, the more easily water passes through it, and therefore the higher the flux value. The permeability of the membrane depends on the thickness of the active layer, porosity and pore size, the nature of the polymer material, and the structural parameter (S) of the supporting layer. The highly permeable membrane reduces water transfer resistance

and improves performance at the same osmotic pressure difference. Even with high permeability, internal ICP may reduce the actual fluxes; therefore, high permeability alone is insufficient if the membrane design is not good [39]. An increased concentration of the feed solution leads to an increase in feed osmotic pressure (π_F) and a decrease in the flux. With increasing feed solution concentration, the ECP effect occurs, whereby the dye accumulates near the membrane surface, reducing the actual osmotic difference more than theoretically expected. The concentration of the draw solution increased leads to an increase in the osmotic pressure at the draw (π_D) and flux. However, actually, the increase in flux is not always linear. Due to ICP within the porous layer of the membrane at very high concentrations of the draw solution, the ICP effect becomes dominant and limits the increase in flux [29].

The performance of NaCl Ds vs flux showed that when the concentration of Ds was experimentally increased from 5 to 70 g/L, the flux was also increased from 3.65 to 9.52 LMH, respectively. This is due to an increase in the effective osmotic pressure (i.e. driving force) for the transport. The result was matched with Al-Alalawy et al [40].

To investigate the behavior of the mathematical model, the modeled water flux using different draw solution concentrations (5, 10, 20, 35, 70 g/L) in the FO system was compared with the experimental results as a function of concentration, as shown in Fig. 4. For the experimental results, each DS concentration was represented by an average flux value. When the draw solution concentration was reduced, there was good agreement between the experimental and theoretical results.

The feed side mass transfer coefficient (k_f) directly affects the water flux value through the phenomenon of ECP. When the mass transfer coefficient (k_f) increased, the ECP decreased, the osmotic pressure at the membrane surface decreased, the effective osmotic difference across the membrane increased, and the flux value increased. The mass transfer coefficient on the draw solution side within the support layer (often denoted as K_D or related to coefficients such as D and S) has a direct and crucial effect on the flux value due to the ICP phenomenon [34]. The draw-side transfer coefficient in the support layer inversely affects internal concentration polarization. Improvement of this coefficient due to reduced ICP, increased osmotic pressure, and increased flux [41].

The large difference between the theoretical and experimental results on water flux when the concentration of the draw solution is increased is not due to an error in the osmotic pressure calculation itself, but rather to fouling, and the calculated permeability is not the real permeability because of the fouling effect. The primary cause is called concentration polarization. This result agreed with Mohammed et al, [34] and phuntsho et al, [36].

Fig. 5 shows that when the real driving force at the membrane sides ($\pi_{D,i} - \pi_{F,m}$) increases from 2.84 to 14.09 bar, the flux also increases from 6.25 to 23.14 LMH and from 3.65 to 9.52 LMH theoretically and

experimentally, respectively. Fig. 6 shows that the draw solution osmotic pressure at the support/active layer interface ($\pi_{D,i}$) from 2.83 to 14.09 bar is lower than the bulk draw solution osmotic pressure ($\pi_{D,b}$) from 4.01 to 56.16 bar because of the DICP effect. This result agreed with Shaoheng et al, [41]

Fig. 7 presents a comparison between the experimental and theoretical results obtained using a 20 ppm MB as feed solution in the FO system. The correlation coefficient (R^2) is 98.89%.

Fig. 8 shows that the recovery of pure water increased from 2.63 to 6.85 with increased concentration of NaCl from 5 to 70 g/L after three hours. In FO, recovery values are typically relatively low compared to RO. Recovery depends on the concentration of the draw solution, membrane permeability, and operating time.

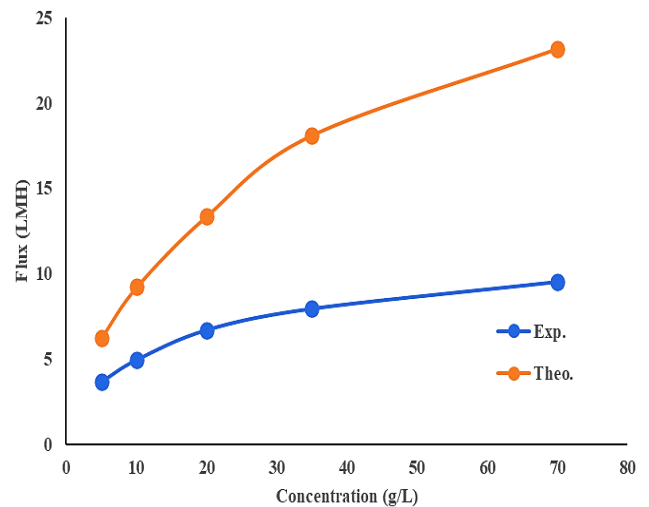


Fig. 4. Experimental and theoretical water flux values vs. concentration of NaCl (20 ppm MB FS with pressure 2 bar)

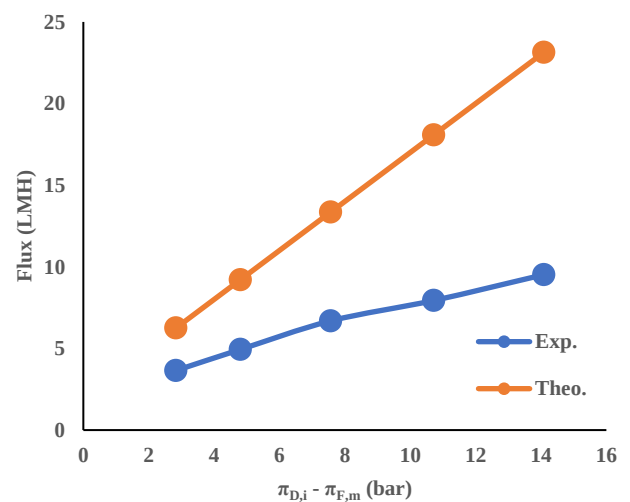


Fig. 5. Effect of the driving force on the water flux theoretically and Experimentally (20 ppm MB FS with pressure 2 bar)

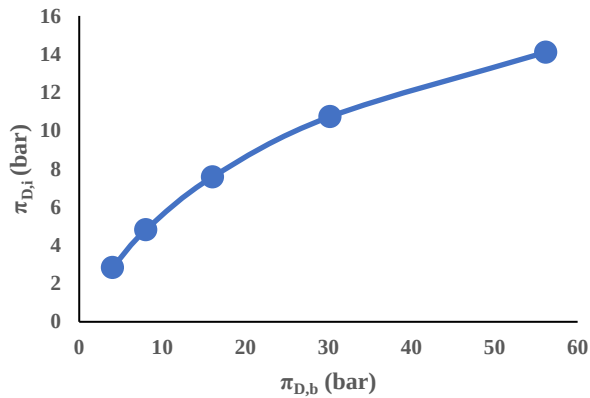


Fig. 6. Bulk osmotic pressure vs. interface osmotic pressure of draw solution

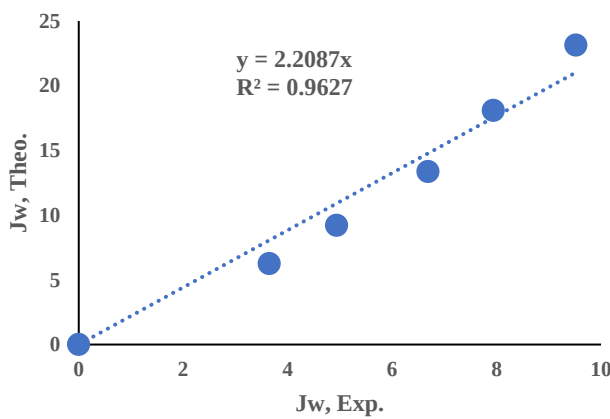


Fig. 7. Comparison of experimental water flux with the theoretical values (Eq. 22) for 20 ppm FS with PF= 2 bar

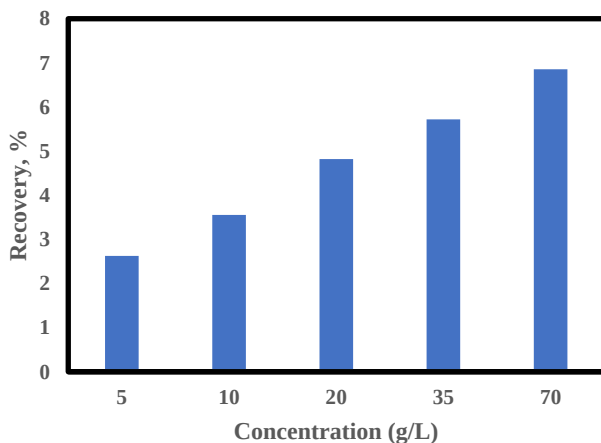


Fig. 8. Recovery of pure water as a function of concentration for simulated MB as FS for 20 ppm with PF= 2 bar

To calculate the theoretical flux of the smart draw solutions, which were prepared in this investigation needs to know the osmotic pressure of these solutions. According to [42] synthesis novel draw solution $\text{K}/\text{Fe}_3\text{O}_4/\text{CNF}$ and able to measure the osmotic pressure for this prepared DS by using an osmometer; the osmotic pressure for $\text{K}/\text{Fe}_3\text{O}_4/\text{CNF}$ in concentrations 0.1 wt/vol%

and 0.075 wt/vol% is equal to 86.1 and 53.8 bar, respectively. The osmotic pressures for the smart draw solutions prepared are expected to fall within this range.

Fig. 9 shows the osmotic pressure of NaCl vs the experimental flux. This Figure can be used to roughly predict the osmotic pressure of these smart DS by plotting the experimental flux value of the prepared DS on the y-axis and reading the osmotic pressure values on the x-axis. For example, the flux value of $\text{K}/\text{Fe}_3\text{O}_4(2\text{g})/\text{MWCNT}$ at 1 to 4 g/L was 8.5, 8.66, 9.06, 9.58 LMH, and the osmotic pressure was 39, 40, 51, and 55 bar, respectively, as previous research has indicated Aende et al, [42].

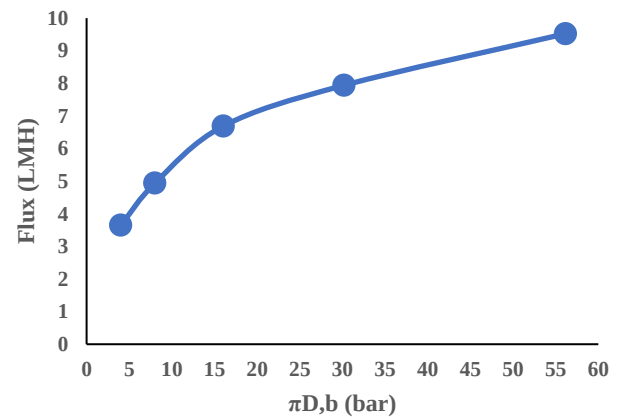


Fig. 9. Effect of the osmotic pressure of NaCl on the water flux experimentally (20 ppm MB FS with pressure 2 bar)

5- Conclusions

This work demonstrates that the mathematical model of the FO process can accurately characterize the water transport behaviour when sodium chloride is used as the draw solution, and methylene blue is used as the feed solution. It showed that the osmotic pressure difference between the two sides of the membrane is the dominant factor in the value of the flux, with its clear influence on the phenomena of internal and external concentration polarization, which were incorporated into the model through mass transfer coefficients and membrane permeability. The use of NaCl has proven to be an effective draw solution due to its high solubility and ability to generate high osmotic pressure, resulting in a significant increase in theoretical and experimental flux.

In contrast, methylene blue, as a feed solution, showed a limited effect on flux due to its large molecular size and low permeability through the membrane, which enhances separation efficiency and reduces unwanted solute migration. The model results also showed good agreement with the expected physical trends, where a higher concentration of the draw solution or improved mass transfer coefficients led to an increase in flux, while a decrease in membrane permeability or an increase in concentration polarization contributed to a decrease in the performance. Accordingly, this mathematical model can be adopted as a reliable tool for predicting, designing, and

optimizing the performance and operating conditions of forward osmosis processes when treating solutions containing organic contaminants such as MB.

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الموديل الرياضي لكفاءة التناضح الأمامي في إزالة صبغة ازرق الميثيلين باستخدام غشاء مركب رقيق الطبقة مسطح TFC

زينب علي إبراهيم^{١,٢,*}، احمد فائق العلوي^١، عبد الله صالح محمد خالد^٣

^١ قسم الهندسة الكيماوية، كلية الهندسة، جامعة بغداد، بغداد، العراق

^٢ الجامعة التقنية الشمالية، الموصل، العراق

^٣ قسم عمليات الهيدروكربونات والبتروكيماويات، كلية الهندسة، جامعة بانونيا، فيسيريم، المجر ٨٢٠٠

الخلاصة

حظيت تقنية التناضح الأمامي (FO) باهتمام بحثي كبير نظراً لتعدد استخداماتها المحتملة في مجالات تحلية المياه وإعادة استخدام مياه الصرف الصحي؛ وتعد هذه التقنية وسيلة فعالة من حيث التكلفة إذ تعتمد على فروق التركيز على جانبي الغشاء دون الحاجة إلى ضغط هيدروليكي خارجي. ويتناول هذا العمل تصميم وبناء نظام للتناضح الأمامي يشتمل على خلية غشائية مثبتة فيها غشاء مركب رقيق الطبقة مسطح من نوع (TFC)، وذلك لإجراء تجارب عملية باستخدام محلول كلوريد الصوديوم (NaCl) كمحلول سحب (DS) وصبغة ازرق الميثيلين (MB) كمحلول تغذية (FS)، ومن ثم استخدام البيانات التجريبية الناتجة لبناء النموذج الرياضي لعملية التناضح الأمامي والتحقق من صحته. تراوحت تراكيز كلوريد الصوديوم (NaCl) بين ٥ و ٧٠ غم/لتر، بينما ثبت تركيز ازرق الميثيلين عند 20 (MB) جزءاً في المليون عند ضغط تغذية قدره ٢ بار. وقد طور نموذج "الذوبان والانتشار" (solution-diffusion model) للتنبؤ نظرياً بتدفق المياه ومقارنته بالنتائج التجريبية باستخدام تراكيز مختلفة لمحلول السحب (DS)، كما دُرِس تأثير ظاهرة الاستقطاب التركيزي. وأظهرت النتائج زيادة في معدل التدفق من ٣,٦٥ إلى ٩,٥٢ لتر/متر مربع/ساعة (LMH)، وارتفاعاً خطياً في الضغط التناضحي عند كتلة المحلول الملامسة للغشاء من ٤,٠١ إلى ٥٦,١٦ بار مع ارتفاع تركيز محلول السحب من ٥ إلى ٧٠ غم/لتر. كما لوحظت زيادة في نسبة استرجاع المياه النقية مع ارتفاع تركيز كلوريد الصوديوم. أظهرت النتائج توافقاً جيداً بين القيم التجريبية والقيم المستمدة من النموذج الرياضي، حيث بلغ معامل الارتباط (R^2) نسبة ٩٨,٨٩%.

الكلمات الدالة: التناضح الامامي، الاستقطاب التركيزي، كلوريد الصوديوم، الموديل الرياضي، الضغط التناضحي.