



Numerical Analysis of Laminar Natural Convection in Square Enclosure with and without Partitions and Study Effect of pPartition on the Flow Pattern and Heat Transfer

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Abstract

The problem of steady, laminar, natural convective flow in an square enclosure with and without partitions is considered for Rayleigh number (10^3 - 10^6) and Prandtl number (0.7). Vertical walls were maintained isothermal at different temperatures while horizontal walls and the partitions were insulated. The length of partition was taken constant. The number of partitions were placed on horizontal surface in staggered arrangement from (1- 3) and ratio of partition thickness ($H/L= 0.033, 0.083, 0.124$). The problem is formulated in terms of the vorticity-stream function procedure. A numerical solution based on a program in Fortran 90 with the finite difference method is obtained. Representative results illustrating the effects of the thickness and number of partitions on the contour maps of the streamlines and temperature are reported and discussed. In addition, the local Nusselt number is evaluated. Results show that the values of stream function (the strength of flow) in enclosure increases with the increasing Rayleigh number. As the number of partitions and thickness of partition increases, the strength of flow decreases. Mean Nusselt number increases with increasing Rayleigh number at different number of partitions. The numerical results are compared with available numerical results and experimental data and a good agreement is obtained.

Keywords: Natural convection, square enclosure, partitioned enclosure.

1- Introduction

Natural convection of fluid media in enclosures has received considerable attention over the past few decades largely due to a wide variety of applications which include building technology, electronic boxes, solar collector technology, energy storage, nuclear reactor technology, etc.

Ben Yedder and Bilgen [1] numerically studied the effect of laminar natural convection in inclined

enclosures bounded by a solid wall for the range $10^3 < Ra < 10^6$ and the inclination angle from 30° to 180° with $Ar=1$ and thermal conductivity ratio K_r varied from 1 to 10. Flow temperature fields and heat transfer rates are examined for these ranges of the Rayleigh number and geometrical parameters of the problem. They found $K_r=10$, which corresponded to the case of high wall conductivity. The

temperature gradient within the solid wall is very small and the temperature at the internal surface is almost the same as the imposed uniform temperature at its outer boundary. For increasing Ra number, the isotherms show a stratified flow within the enclosure with steep gradients near the vertical boundaries.

Using FVM, Miomir Raos and Ljiljana Ivković [2] numerically investigated the laminar natural convection phenomena in two-dimensional rectangular enclosure with differentially heated sides and adiabatic horizontal walls, and the effect of rotation of the enclosure is presented in this study too. The study assumed $\text{Pr}=0.7$ and Rayleigh number from 10^3 to 10^6 . The results showed complex flow patterns heat transfer rates, with different orientation of the enclosure. Angle of rotation about 65-75 maximizes Nu number value for named conditions.

Nuri Yucel and Hakan Ozdem [3] studied natural convection heat transfer in partially divided square enclosures. A finite difference computer program based on control volume approach is developed for the solution.

The study assumed $\text{Pr}=0.7$ and Rayleigh number from 10^3 to 10^6 . Vertical side walls are kept at different constant temperature, while the horizontal walls are insulated. The effects of Ra number and number of partitions on heat transfer are investigated. They found that, the heat transfer rate increases with increasing Ra number, and at low Ra numbers, the conduction is the dominant heat transfer mode, and at $\text{Ra}=10^3$; the mean Nusselt number remains constant around unity for all numbers of partitions.

Dias and Milanez [4] numerically studied a 3-D laminar natural convection in air filled enclosure using finite volume technique .The results

were obtained for Ra number that ranges from (10^3 to 10^6) and aspect ratios ranging from (1 to 20).They found that the two dimensional approximation, frequently compared to experimental results, deviate from the three dimensional results as the Rayleigh number increases.

Khudheyer S. Mushatet [5] numerically investigated laminar natural convection inside a rectangular cavity containing two cylindrical obstacles. A curvilinear coordinates system was used to transfer the physical space into a computational domain. The governing partial differential equations are solved using stream function and vorticity method. The vorticity and energy equations were solved by using an alternate difference scheme(ADI) while the stream function with an iteration method. The cavity was differentially heated. The effect of the distance between the obstacles was tested for Rayleigh number range $10^3 \leq \text{Ra} \leq 10^5$.The documented results show that the fluid flow and temperature fields significantly depend on the distance between the obstacles for the studied Rayleigh numbers.

Nienchuan and Adrian [6] experimentally and analytically studied the phenomenon of heat transfer by natural convection in a partially divided enclosure. The nonconducting partition was fitted to the floor of cavity. Heat transfer measurements and flow visualization studies were conducted in Rayleigh number range 10^9-10^{10} for aperture ratios (height of the internal opening above the partition: height of the enclosure) of 1, 1/4, 1/8, 1/16 and 0. It was found that, as the aperture ratio decreases from 1 to 0, the Nusselt number decreases by a factor of 15.

Diego Angelia et al.[7] presented numerical analysis of transitional natural convection from a confined

thermal source. The system considered is an air-filled, square-sectioned 2D enclosure containing a horizontal heated cylinder. The resulting flow is investigated with respect to the variation of the Rayleigh number, for three values of the aspect ratio A. The first bifurcation of the low-Ra fixed-point solution is tracked for each A-value. Chaotic flow features are detailed for the case A = 2.5. The supercritical behaviour of the system is investigated using nonlinear analysis tools and phase-space representations, and the effect of the flow on heat transfer is discussed.

Scozia and Frederich [8], and Yucel and Turkoglu [9] studied natural convection in vertical rectangular cavities with multiple fins attached to the one of the active walls, numerically. They observed that the heat transfer rate increased with the increasing number of fins and the fin length at low Rayleigh numbers; the heat transfer rate can be decreased or increased by properly choosing the number of fins and the fin lengths.

There are several experimental and numerical studies on natural convection heat transfer in enclosures; however, studies about the partially divided enclosures are rare especially, for study effect of thickness of partition.

In this study, natural convection of air with the Prandtl number of 0.7 was analyzed for two different cases as follows:

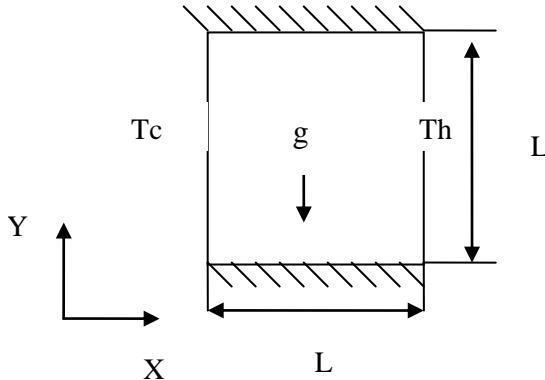
Case-1- without dividers

Case-2- with different number of dividers placed on horizontal surfaces in staggered arrangement and change thickness of dividers to show how the effect of various dividers on the flow pattern and heat transfer.

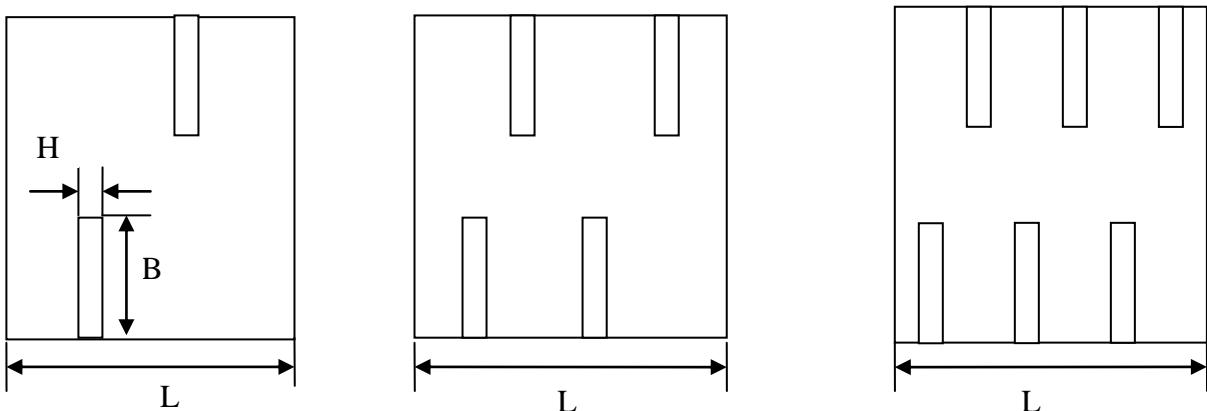
2- Physical Model

The problem studied in this paper is a two dimensional laminar flow and heat transfer in a square enclosure. The considered Rayleigh numbers was ranged from (10^3 to 10^6). The working fluid was air with $\text{Pr}=0.70$. Vertical walls were maintained isothermal at different temperatures while horizontal walls and the partitions were placed on horizontal surface in staggered arrangement insulated. The length of partition was taken constant, but thickness of partitions is represented by ratio ($H/L= 0.033, 0.083, 0.124$) and the number of dividers changes from (1-3) as shown in Fig.(1).

The change of the streamlines and isotherms are obtained numerically and presented graphically. Also the change of the mean Nusselt numbers along the hot walls is calculated for different cases.



(a) Case -1- without dividers



(b) Case -2- with different number of dividers

Fig. 1, Shematic of problem and the coordinate system for two cases (a ,b)

3- Mathematical Model

The fluid is Newtonian and the flow is steady, laminar, two dimensions and incompressible. The non-dimensional governing equations for the conservation of mass, momentum and energy are of Abdullatif and Ali [10]:

Continuity:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$

X- momentum:

$$\begin{aligned} \frac{\partial}{\partial X}(UU) + \frac{\partial}{\partial Y}(VU) &= -\frac{\partial P}{\partial X} + \\ \frac{1}{Re} \left[\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right] \end{aligned} \quad (2)$$

Y- momentum:

$$\begin{aligned} \frac{\partial}{\partial X}(UV) + \frac{\partial}{\partial Y}(VV) &= -\frac{\partial P}{\partial Y} + \\ \frac{1}{Re} \left[\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right] + Ra Pr \theta \end{aligned} \quad (3)$$

Energy:

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \left[\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right] \quad (4)$$

With

$$\begin{aligned} U &= \frac{u}{u_\infty}, \quad V = \frac{v}{u_\infty}, \quad X = \frac{x}{L} \\ Y &= \frac{y}{L}, \quad P = \frac{p}{\rho u_\infty^2}, \quad \theta = \frac{T - T_c}{T_h - T_c} \\ Pr &= \frac{\nu}{\alpha}, \quad Ra = \frac{g \beta (T_h - T_c) L^3}{\alpha \nu} \end{aligned}$$

The governing equations can be written in dimensionless stream function–vorticity form as:

$$\frac{\partial}{\partial X}(U\omega) + \frac{\partial}{\partial Y}(V\omega) = \text{Pr} \left[\frac{\partial^2 \omega}{\partial X^2} + \frac{\partial^2 \omega}{\partial Y^2} \right] + Ra \text{Pr} \frac{\partial \theta}{\partial X} \quad (5)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \quad (6)$$

The only non-zero component of the vorticity is:

$$\omega = \frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y} \quad (7)$$

From the definition of stream function which verify the continuity equation, vertical and horizontal components can be written as:

$$V = -\frac{\partial \psi}{\partial X} \quad (8)$$

$$U = \frac{\partial \psi}{\partial Y} \quad (9)$$

By substituting equation (8) and (9) into equation (7) to obtain the following stream equation:

$$-\omega = \frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = \Delta^2 \psi \quad (10)$$

4- Boundary Conditions

The appropriate initial and boundary conditions for the problem under consideration can be written in dimensionless form as:

1. The left wall is ($X=0$):

$$U = 0, V = 0, \psi = 0, \theta = 0,$$

$$\omega = -\left(\frac{\partial^2 \psi}{\partial X^2} \right)$$

2. The right wall is ($X=1$):

$$U = 0, V = 0, \psi = 0, \theta = 1,$$

$$\omega = -\left(\frac{\partial^2 \psi}{\partial X^2} \right)$$

3. Insulated walls: the lower wall is ($Y=0$) and upper wall is ($Y=1$):

$$U = 0, V = 0, \psi = 0, \frac{\partial \theta}{\partial Y} = 0, \omega = -\left(\frac{\partial^2 \psi}{\partial Y^2} \right)$$

5- Numerical Solution

The numerical solution of this system may be obtained by solving its difference system with some available iteration procedure. Consider the second order conservative, monotonic finite difference scheme approximating system of equations (5), (6) and (10). The scheme was used for steady convective problems involving wide ranges of process parameters and gave good results. It employed the integrointerpolation method. A system of difference equations is obtained by integrating the original system (5), (6) and (10) Nogotov [11].

Following Nogotov's procedure [11] the governing finite difference equations for (ω , θ , and ψ) can be written in the standard five point formula form. These finite difference equations which subject to appropriate boundary conditions are solved by an iterative method known as successive substitution. If (ω^s , θ^s , and ψ^s) denote functional values at the end of s th iteration, the value of (ω , θ , and ψ) at ($s+1$)th iteration level are calculated from the following expressions:

$$\begin{aligned} \theta_{i,j}^{s+1} &= (1 - F_\theta) \theta_{i,j}^s + \frac{F_\theta}{A_\theta} (a_\theta \theta_{i+1,j}^s + b_\theta \theta_{i-1,j}^s \\ &\quad + c_\theta \theta_{i,j}^s + d_\theta \theta_{i,j-1}^{s+1}) \end{aligned} \quad (11)$$

$$\begin{aligned} \omega_{i,j}^{s+1} &= (1 - F_\omega) \omega_{i,j}^{s+1} + \\ &\quad \frac{F_\omega}{A_\omega} \left[(a_\omega \omega_{i+1,j}^s + b_\omega \omega_{i-1,j}^{s+1} + c_\omega \omega_{i,j+1}^s + d_\omega \omega_{i,j-1}^{s+1}) \right. \\ &\quad \left. + 0.5 Ra \text{Pr} h (\theta_{i+1,j}^{s+1} - \theta_{i-1,j}^{s+1}) \right] \end{aligned} \quad (12)$$

$$\begin{aligned}\psi_{i,j}^{s+1} &= (1 - F_\psi) \psi_{i,j}^s + \\ &\frac{F_\psi}{4} (\psi_{i+1,j}^s + \psi_{i-1,j}^{s+1} + \psi_{i,j+1}^s + \psi_{i,j-1}^{s+1} + h^2 \omega_{i,j-1}^{s+1})\end{aligned}\quad (13)$$

Where (s) is the iteration number, and F_θ, F_ω and F_ψ are the relaxation parameters which depend on the mesh size and fluid mechanical parameters.

A converged solution was defined as one that meet the following criterion for all dependent variables.

$$\max \left| \frac{\phi^{n+1} - \phi^n}{\phi^{n+1}} \right| \leq 10^{-5}$$

6- Calculation of Mean Nusselt Number

The mean Nusselt number can be calculated from equation below Nogotov's equations [11]:

$$\overline{Nu} = \int_0^1 \left. \frac{\partial \theta}{\partial Y} \right|_{Y=0} dX \quad (14)$$

The integration can be evaluated by using numerical integration (Simpson's rule) to obtain an overall Nusselt number as below Nogotov [11]:

$$\overline{Nu} = \frac{h}{3} \left[Nu_{L(1)} + 4 \sum_{j1} Nu_{L(j1)} + 2 \sum_{j2} Nu_{L(j2)} + Nu_{L(n)} \right] \quad (15)$$

7- Results and Discussions

Numerical results for the steady-state streamline and temperature contours within the partitioned enclosure for various values of the Rayleigh number.

Fig.(2) shows effects of Ra number on the stream function and temperature contours for partition thickness ($H/L=0.0$) and number of partitions (0). For small Rayleigh numbers ($Ra=10^3$), the free convection currents are small and the values of stream function in enclosure increases with the increasing

Rayleigh number. The temperature contours within the enclosure are almost for different Rayleigh number.

In Fig.(3) the streamline and isotherm contours for different Rayleigh numbers are plotted for the number of partitions: (1) and thickness of ($H/L = 0.033$).The values of stream function (the strength of flow) in enclosure increases with the increasing Rayleigh number. At low Rayleigh numbers, there is only one circulating cell. Increasing Rayleigh numbers, two circulating cells form inside the enclosure; with further increase in Rayleigh numbers, the number of circulating cells formed in the enclosure becomes three. it is also seen that the values of stream function decreases with increasing number of partition in Figs.(4,5)

A similar trend were observed for partition thickness ($H/L= 0.083, 0.124$) in Figs. (6,7 and 8) and Figs. (9, 10 and 11).

To analyze the effects of the partition thickness on the temperature contours, the computations are performed for different partition thickness at different Rayleigh number. in Figs. (3 to 11), it is observed no significant difference can be seen on temperature contours for different partition thickness or number of partitions.

The variation of mean Nusselt number with Rayleigh number is shown in Fig.(12). As seen, the mean Nusselt number increases with increasing Rayleigh number. It is also seen that variation of mean Nusselt number decreases with increasing number of partitions. As the number of partitions increases, partitions partially block the motion of fluid flow and as a result, the effect of convection decreases. A similar behavior for dimensionless partition thickness at ($H/L=0.083, 0.124$)

In Fig. (13), the variation of the mean Nusselt number with number of

partitions is shown for different Rayleigh numbers. At higher Rayleigh numbers, the mean Nusselt number decreases with increasing number of partitions. The reason is that the increase in the number of partitions retards the fluid flow and decreases the circulation rate.

The dimensionless partition thickness at ($H/L=0.083, 0.124$) is also similar to that of the dimensionless partition thickness at ($H/L=0.033$).

The numerical results of the present work are compared with available numerical results and experimental data. As displayed in Fig. (14), the present simulation shows good agreement with the numerical data for Nuri Yucel and Ahmed Hakan Ozdem [3.]

8- Conclusion

- 1- The values of stream function (the strength of flow) in enclosure increases with the increasing Rayleigh number.
- 2- Mean Nusselt number increases with increasing Rayleigh number at different number of partitions.
- 3- As the number of partitions and thickness of partition increases, the strength of flow decreases.
- 4- Divided square enclosures is strongly affected in staggered arrangement.
- 5- Dimensionless partition thickness (H/L) has no effect on mean Nusselt number.

9- Nomenclature

Symbol	Description	Unit
B	Height of the partition	m
g	Gravitational acceleration	m/sec^2
H	Thickness of the partition	m
L	Enclosure total length	
Nu	Mean Nusselt number	
Pr	Prandtl number ($\text{Pr} = \nu/\alpha$)	
Ra	Rayleigh number $\text{Ra} = g\beta(T_h - T_c)L^3/\nu\alpha$	
T	Temperature	K
T_c	Cold wall temperature	K
T_h	Hot wall temperature	K
U	Dimensionless velocity component in x-direction	
V	Dimensionless velocity component in y-direction	
x	Horizontal axis	m
X	Dimensionless horizontal axis ($X = x/L$)	
y	Vertical axis	m
Y	Dimensionless vertical axis ($Y = y/L$)	
ν	Kinematic viscosity	m^2/sec
θ	Dimensionless temperature ($\theta = (T - T_c)/(T_h - T_c)$)	
ψ	Stream function	
ω	Vorticity	
β	Volume coefficient of expansion	1/K
α	Thermal diffusivity ($\alpha=k/\rho cp$)	m^2/sec

10- References

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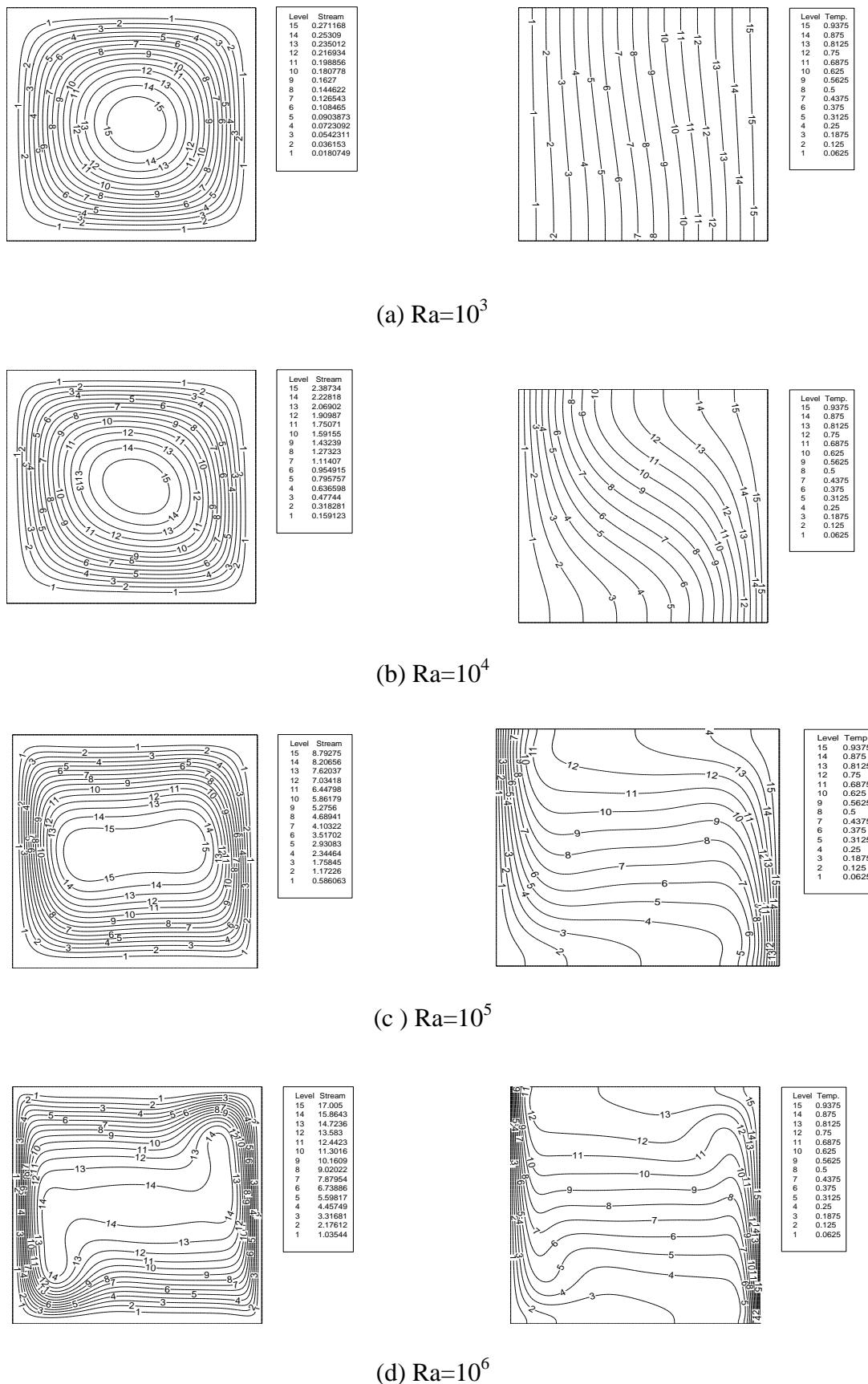


Fig. 2, Effects of Ra number on the stream function and temperature contours for partition thickness ($H/L= 0.0$) and number of partitions (0)

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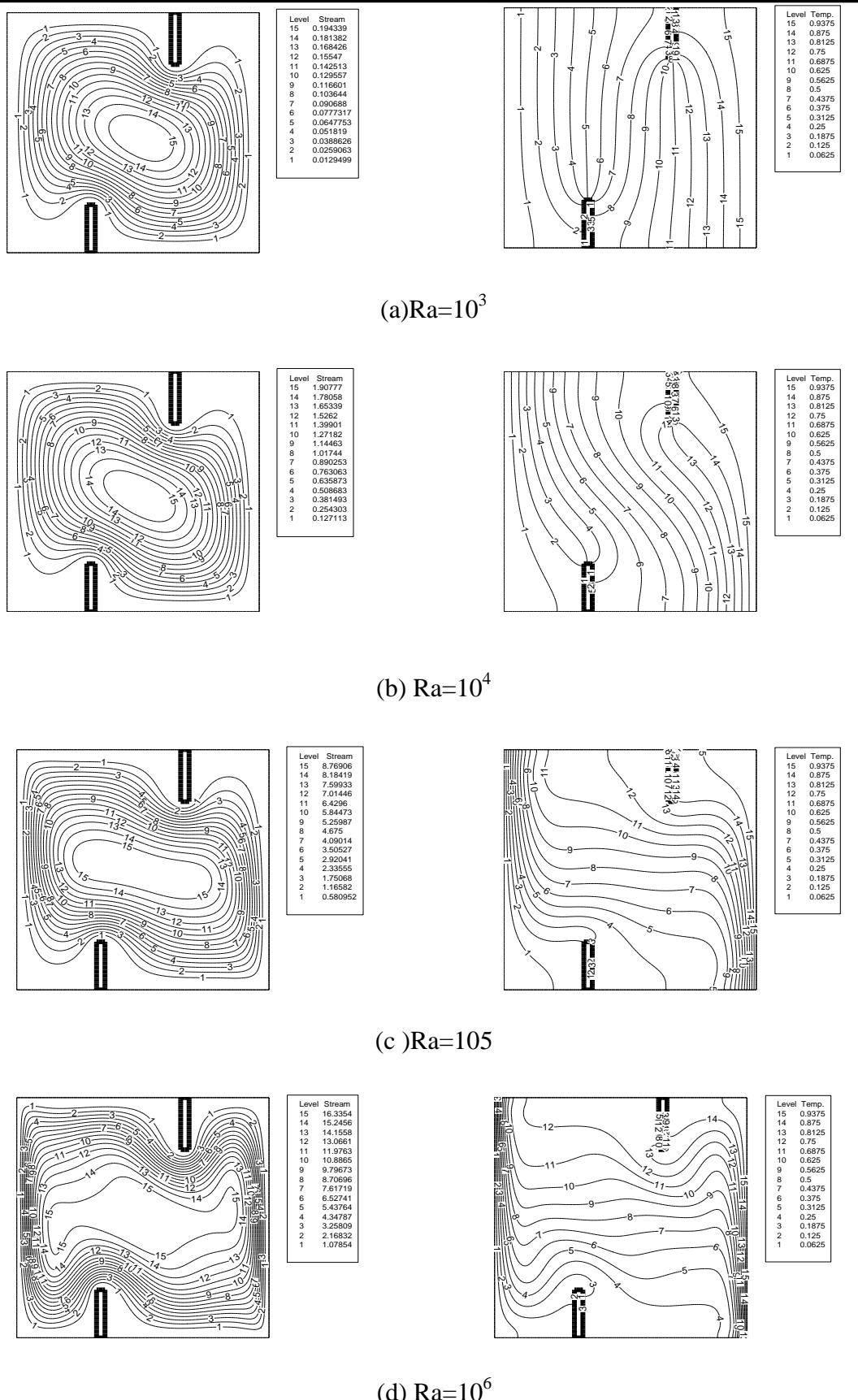


Fig. 3, Effects of Ra number on the stream function and temperature contours for partition thickness ($H/L = 0.033$) and number of partitions (1)

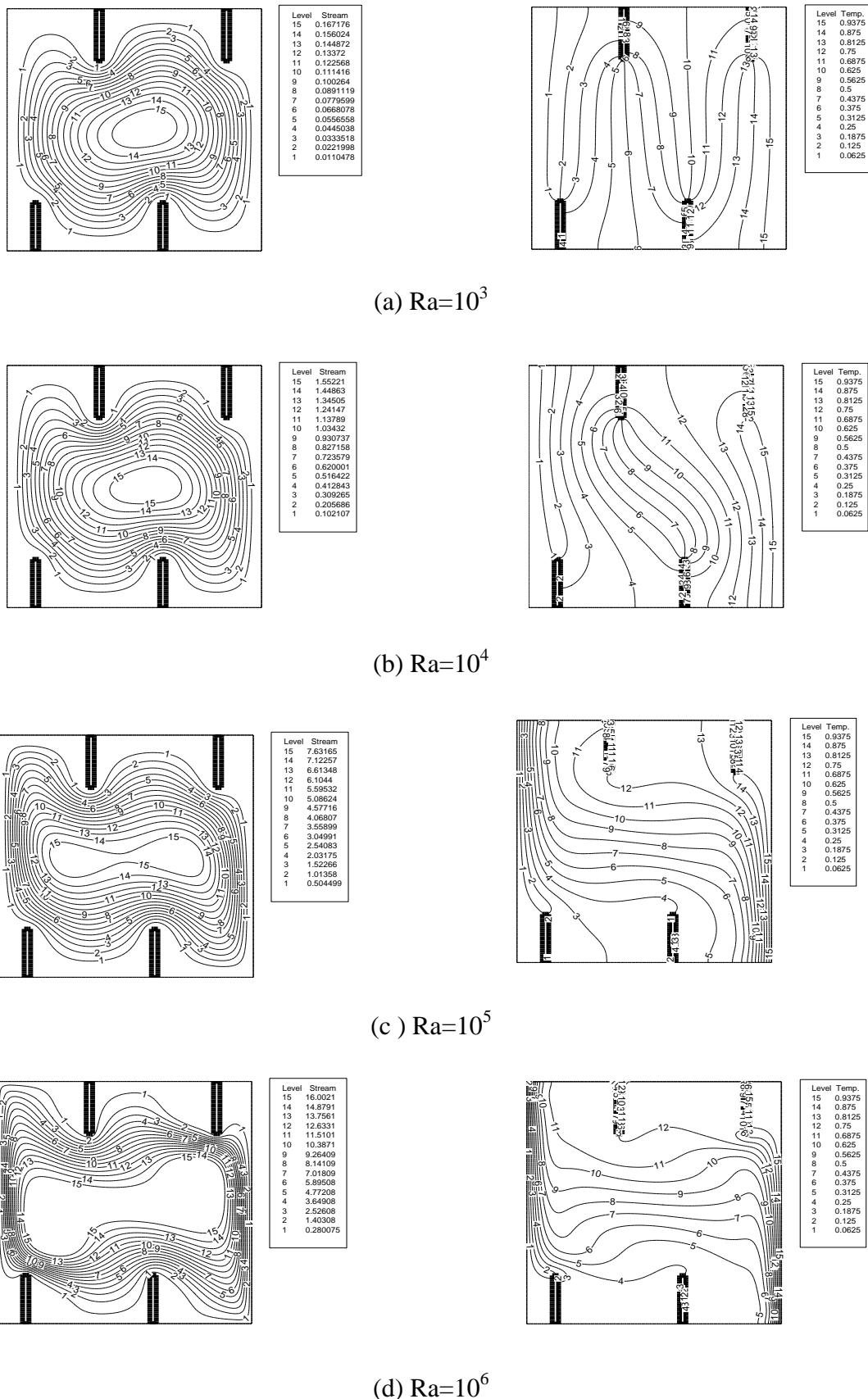


Fig. 4, Effects of Ra number on the stream function and temperature contours for partition thickness ($H/L = 0.033$) and number of partitions (2)

Numerical Analysis of Laminar Natural Convection in Square Enclosure with and without Partitions and Study Effect of pPartition on the Flow Pattern and Heat Transfer

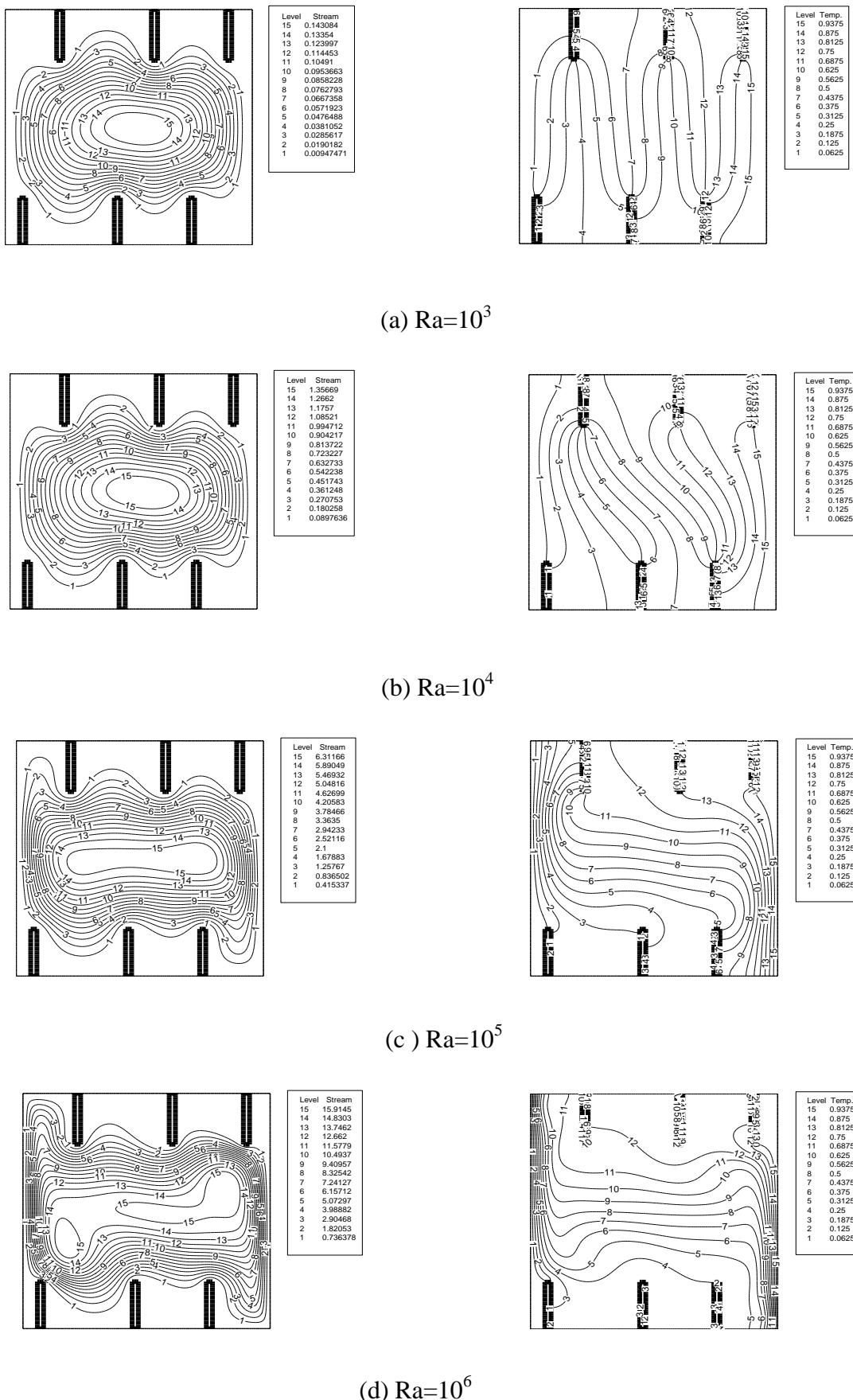


Fig. 5, Effects of Ra number on the stream function and temperature contours for partition thickness ($H/L= 0.033$) and number of partitions (3)

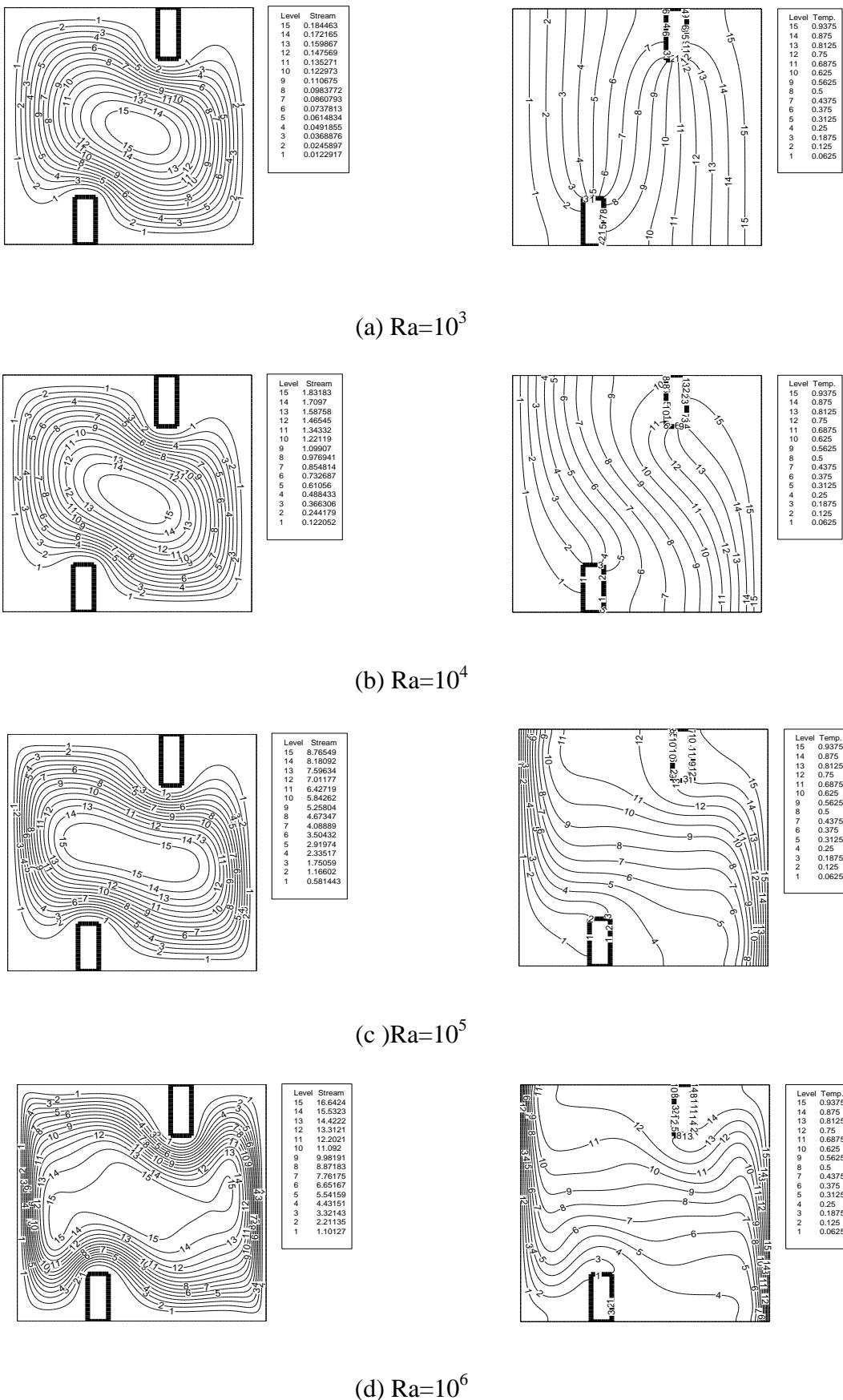


Fig. 6, Effects of Ra number on the stream function and temperature contours for partition thickness ($H/L = 0.083$) and number of partitions (1)

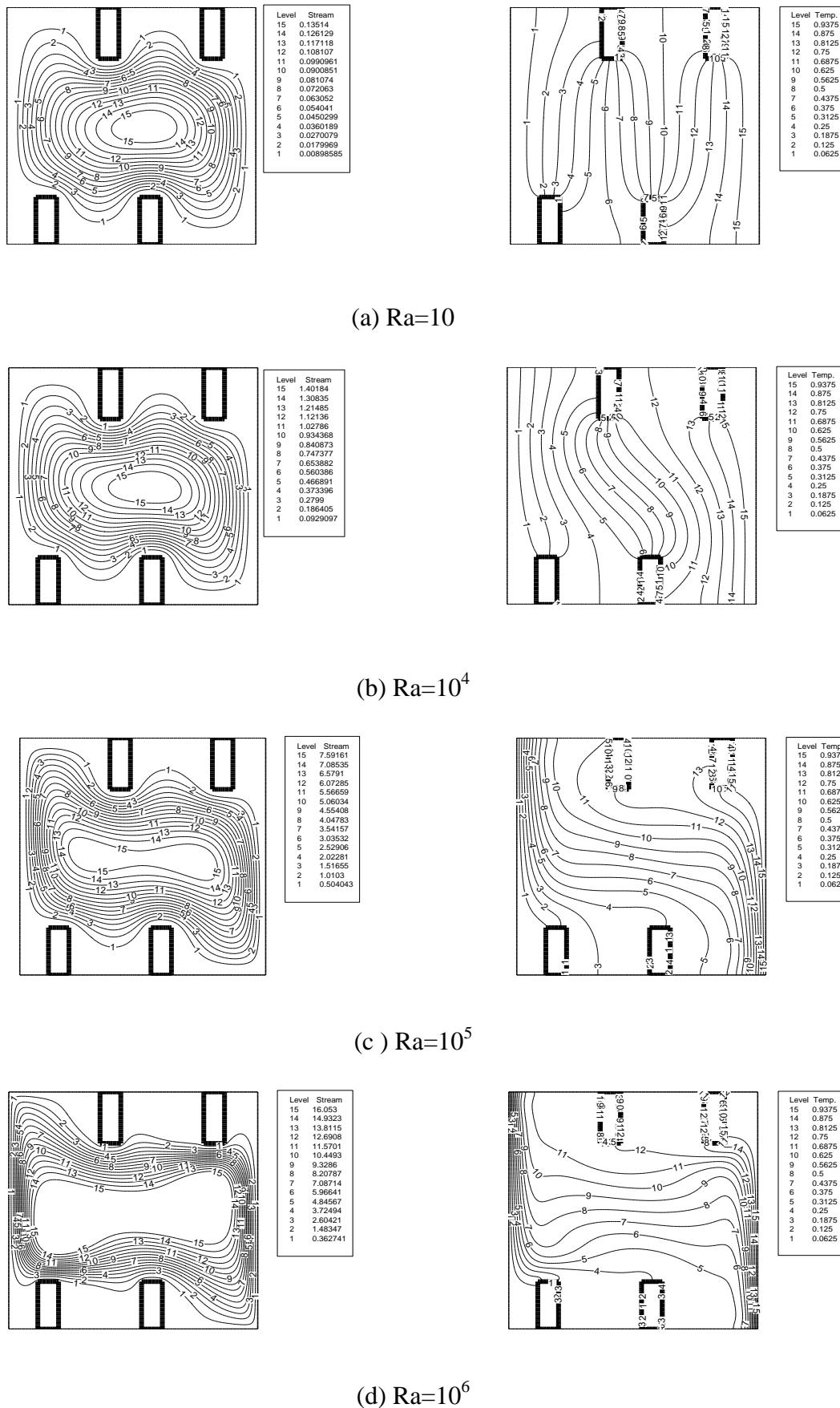


Fig. 7, Effects of Ra number on the stream function and temperature contours for partition thickness ($H/L= 0.083$) and number of partitions (2)

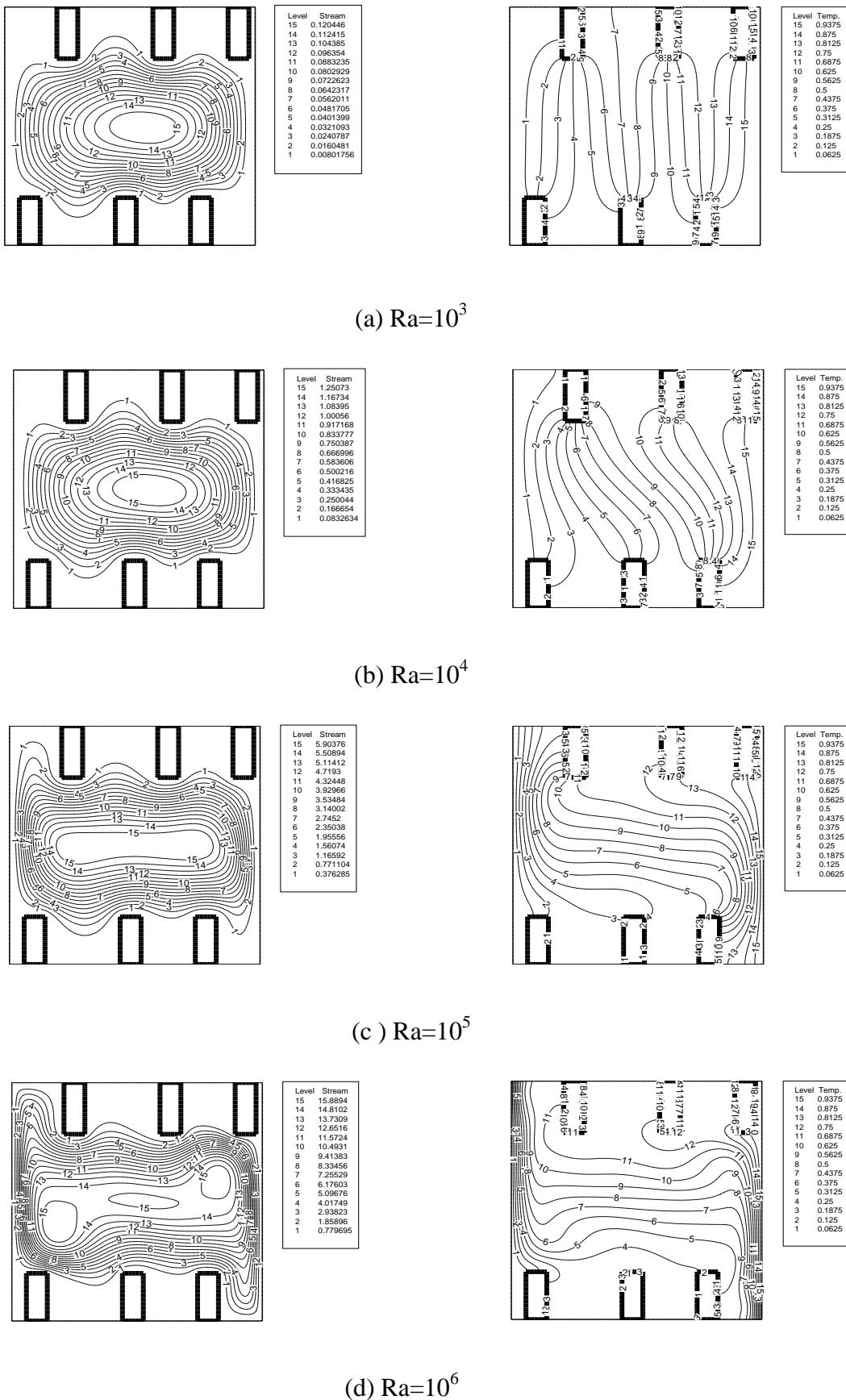


Fig. 8, Effects of Ra number on the stream function and temperature contours for partition thickness ($H/L = 0.083$) and number of partitions (3)

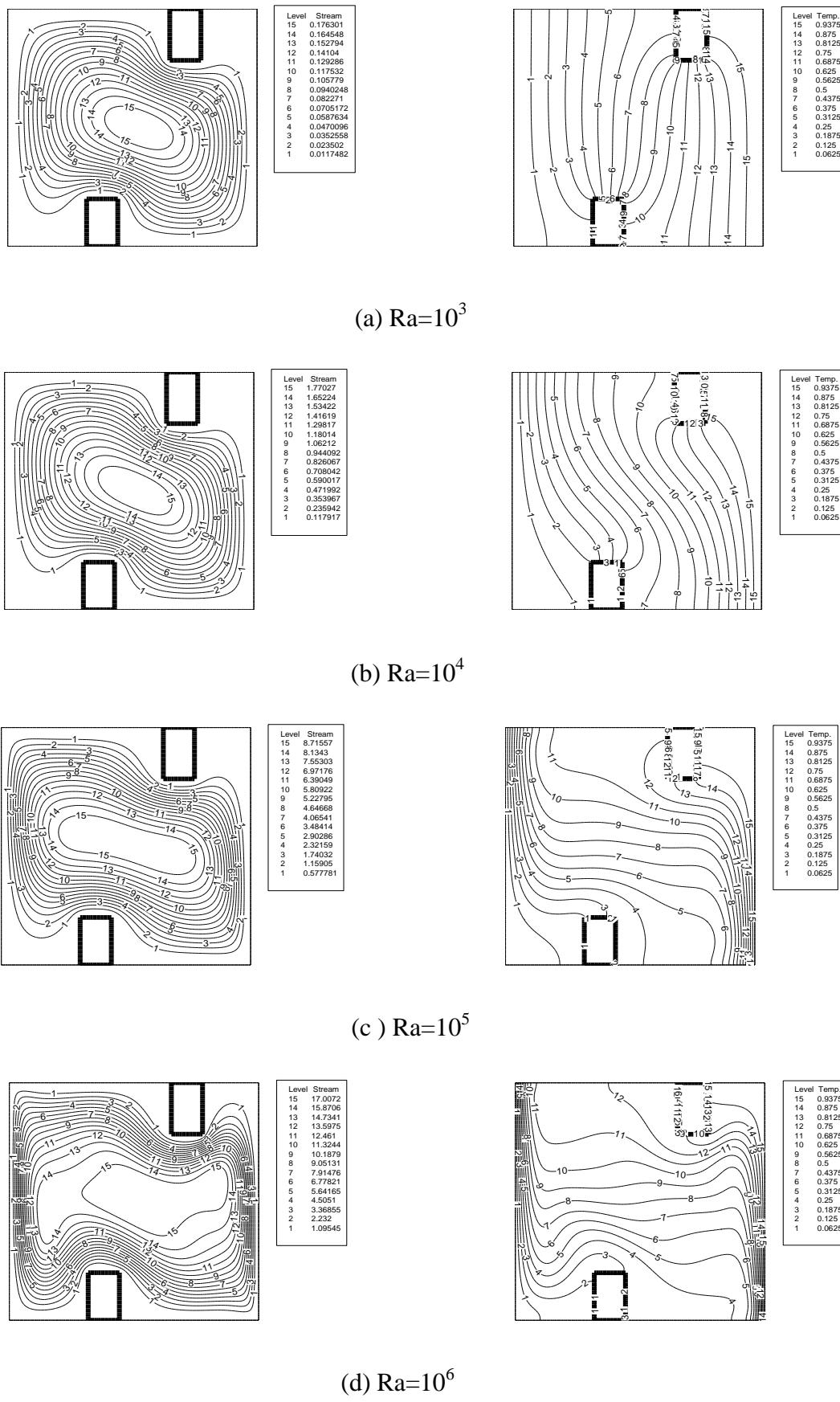


Fig. 9, Effects of Ra number on the stream function and temperature contours for partition thickness ($H/L = 0.124$) and number of partitions (1)

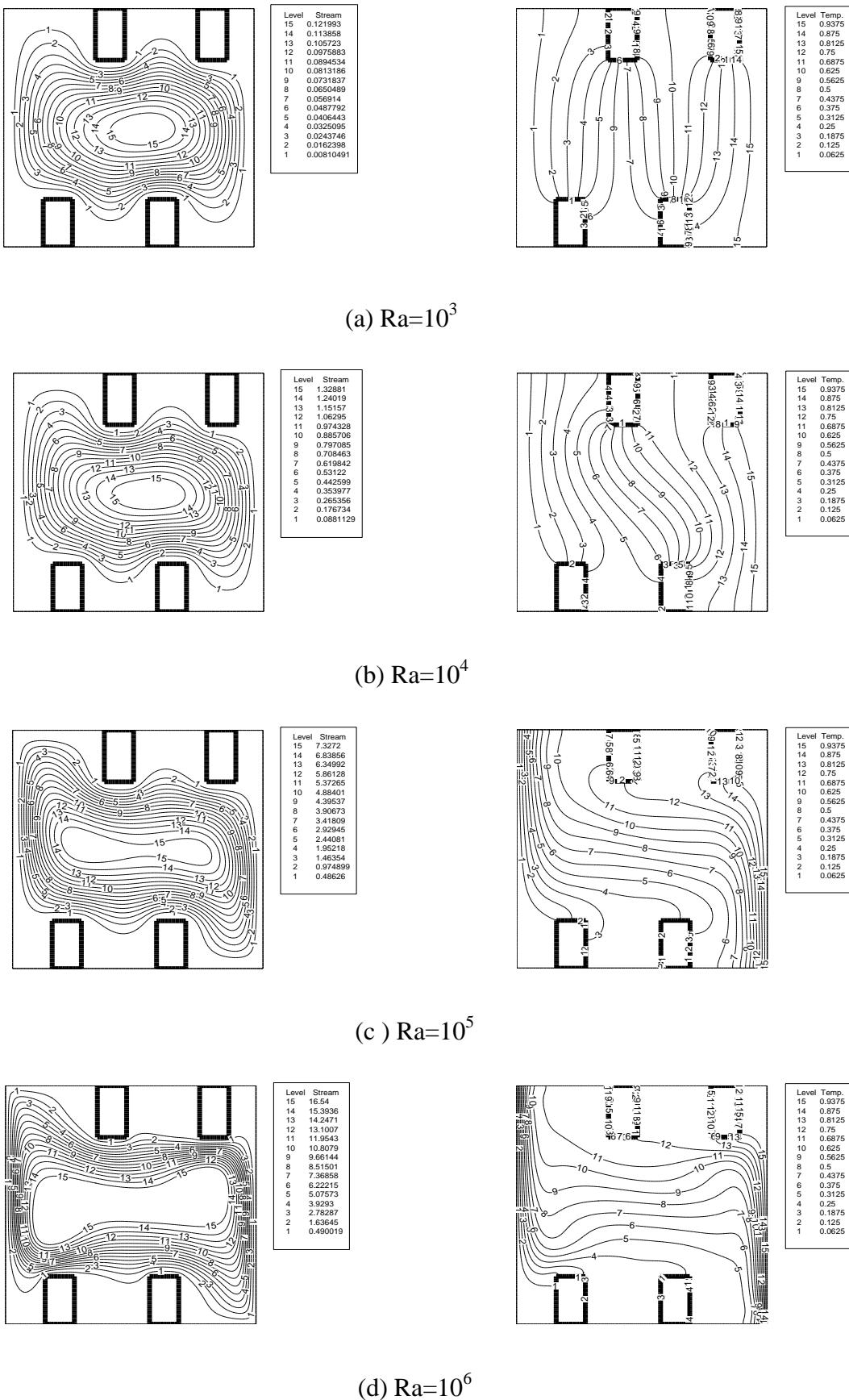


Fig. 10, Effects of Ra number on the stream function and temperature contours for partition thickness ($H/L = 0.124$) and number of partitions (2)

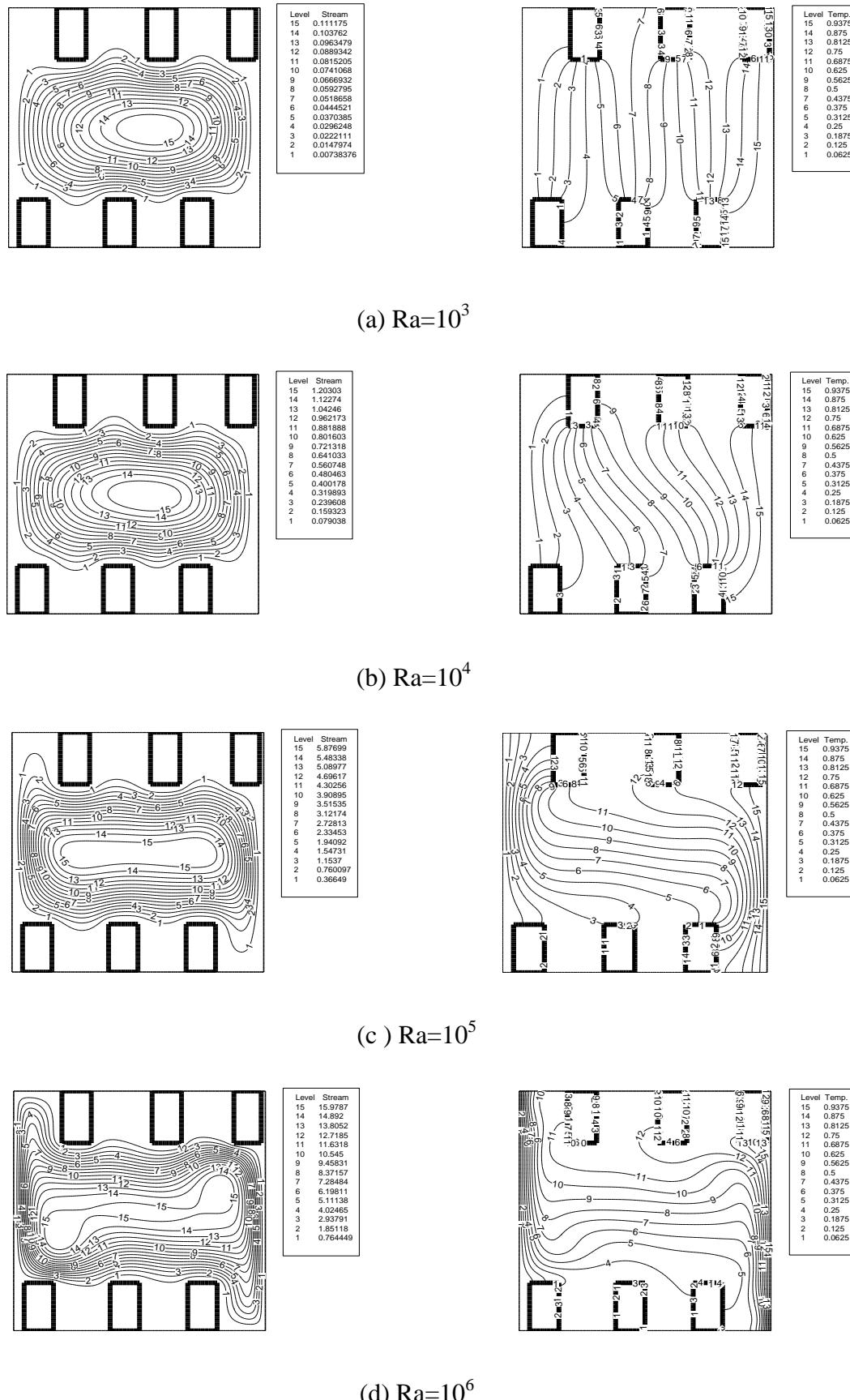


Fig. 11, Effects of Ra number on the stream function and temperature contours for partition thickness ($H/L = 0.124$) and number of partitions (3)

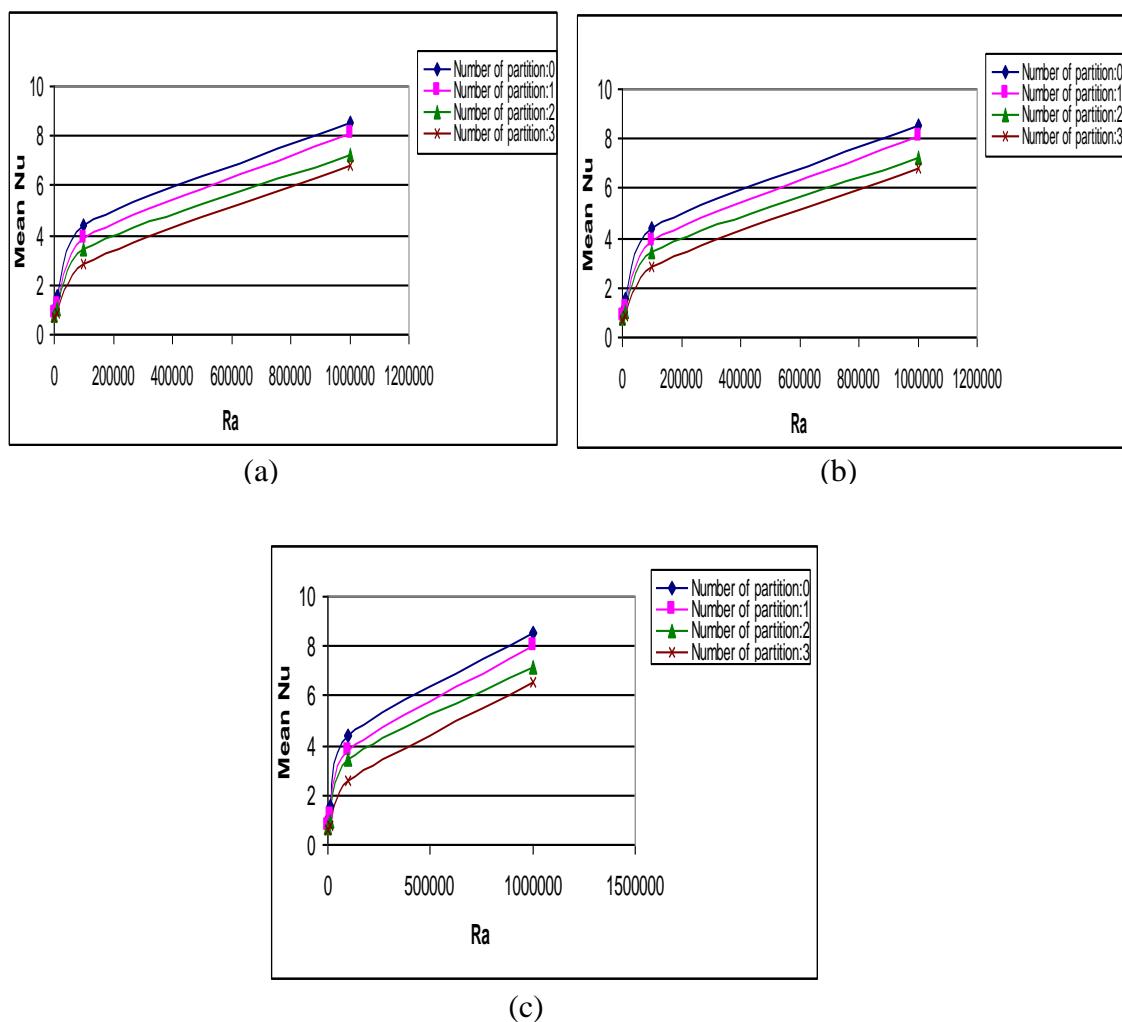


Fig. 12, Variation of mean Nusselt number with Rayleigh number at different number of partitions for dimensionless partition thickness (a-H/L=0.033), (b-H/L=0.083), (c-H/L=0.124)

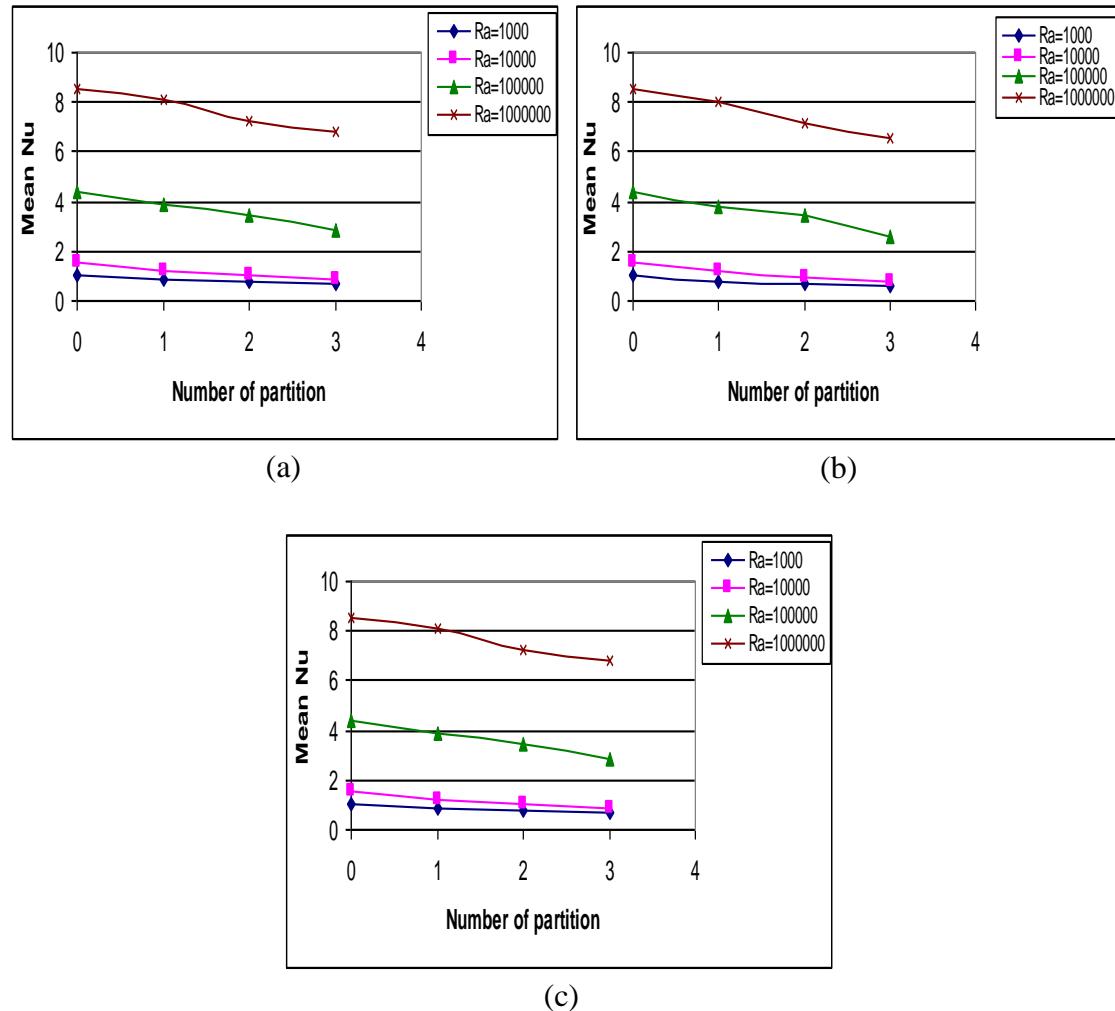
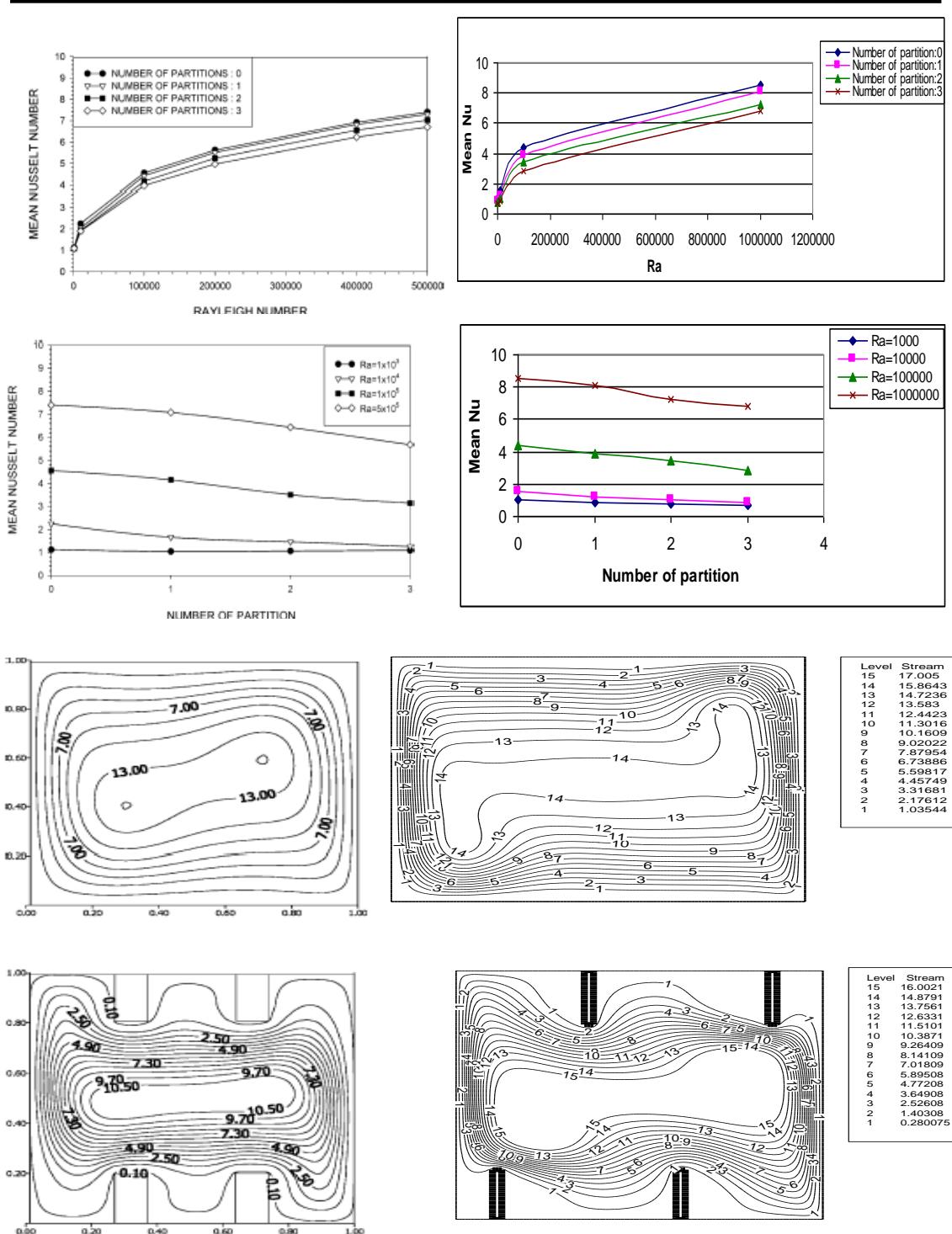


Fig. 13, Variation of number of partitions with mean Nusselt numbers at different Rayleigh numbers for dimensionless partition thickness (a- $H/L=0.033$), (b- $H/L=0.083$), (c- $H/L=0.124$)



Nuri Yucel and Ahmed Hakan Ozdem [3.]

Present work

Fig. 14, Comparison of the present work with that of Nuri Yucel, and Ahmed Hakan Ozdem [3] for $\text{Ra}=10^6$

الخلاصة

يتضمن البحث الحالي دراسة نظرية عددية لانقال الحرارة بالحمل الحر للجريان المستقر داخل حيز مربع بوجود او عدم وجود الحاجز. الدراسة تغطي المدى لرقم رايلي من 10^3 الى 10^6 ولعدد برانتل ($Pr=0.7$). الجدران العمودية للحيز ذات درجات حرارة ثابتة لكن مختلفة بينما الجدران الافقية معزولة. ارتفاع الحاجز الموضوعة داخل الحيز ثابت وموضوعة على الجدران الافقية بترتيب مخالف وبأعداد مختلفة تتراوح من (1 - 3) وسمك الحاجز المستخدم تساوي ($H/L=0.033, 0.083, 0.124$). الحل العددي تم باستخدام طريقة دالة الانسياب الدوامية باستخدام طريقة الفروق المحددة مع بناء برنامج حاسوبي بلغة (فورتران 90). تم دراسة تاثير كل من رقم رايلي ، عدد الحاجز ، وسمك الحاجز على شدة الدوامات الدوارة و انقال الحرارة داخل الحيز . بالإضافة الى ذلك تم ايجاد معدل قيم رقم نسلت. بينت النتائج ان شدة الدوامات الدوارة تزداد مع زيادة عدد رايلي بينما تقل عند زيادة عدد الحاجز او سماك الحاجز . كذلك بينت النتائج ان متوسط نسلت يزداد مع زيادة رقم رايلي. وتم مقارنة النتائج العددية مع النتائج العملية المتوفرة وكانت نتائج المقارنة جيدة.