

Iraqi Journal of Chemical and Petroleum Engineering Vol.13 No.3 (September 2012) 35-45 ISSN: 1997-4884



# The Control of Non Isothermal CSTR Using Different Controller Strategies

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#### Abstract

In all process industries, the process variables like flow, pressure, level, concentration and temperature are the main parameters that need to be controlled in both set point and load changes.

A control system of propylene glycol production in a non isothermal (CSTR) was developed in this work where the dynamic and control system based on basic mass and energy balance were carried out.

Inlet concentration and temperature are the two disturbances, while the inlet volumetric flow rate and the coolant temperature are the two manipulations. The objective is to maintain constant temperature and concentration within the CSTR.

A dynamic model for non isothermal CSTR is described by a first order plus dead time (FOPDT).

The conventional PI and PID control were studied and the tuning of control parameters was found by Ziegler-Nichols reaction curve tuning method to find the best values of proportional gain ( $K_c$ ), integral time ( $\tau_I$ ) and derivative time ( $\tau_D$ ).

The conventional controller tuning is compared with IMC techniques in this work and it was found that the Ziegler –Nichols controller provides the best control for the disturbance and the worst for the set-point change, while the IMC controller results show satisfactory set-point responses but sluggish disturbance responses because the approximate FOPTD model has relatively small time delay.

Feedforward and feedforward combined with feedback control systems were used as another strategy to compare with above strategies. Feedforward control provides a better response to disturbance rejection than feedback control with a steady state deviation (offset). Thus, a combined feedforward-feedback control system is preferred in practice where feedforward control is used to reduce the effects of measurable disturbances, while feedback trim compensates for inaccuracies in the process model, measurement error, and unmeasured disturbances. Also the deviation (offset) in feedforward control was eliminated.

**Keywords:** Non isothermal CSTR, Ziegler –Nichols reaction curve, IMC control and Feed forward control.

## Introduction

The non-isothermal CSTR is an important industrial process that introduces the opportunity for a diverse

range of process dynamics. This work involves the control of the production of propylene glycol by the hydrolysis of propylene oxide. Feedback controller settings were calculated using the Ziegler-Nichols and IMC tuning. Different control strategies were studies; these strategies are feedback, Feedforward and Feedforward with feedback control.

The selection of good control algorithm depends upon the performance comparison of different possible control techniques and selecting the best for the desired condition.

A control system designed for a particular process should provide fast and accurate changes for both the set point changes as well for load changes. Model based controllers are now popular because of their ability to handle a process with dead time. One type of model based control is Internal Model Control (IMC) which has both an open loop and a closed loop system. IMC tuning is referred to as a set of tuning procedures based on the internal model principle. The underlying idea behind internal model methodologies is to compute a controller and/or to set its values relative to a prescribed response formulated as a prescribed (internal) model. In this way, IMC designs belong to the class of model based control settings, whose origin can be traced back to the Proportional-Integral-Derivative (PID) tuning method proposed by Dahlin [1].

Feedforward control strategies was studied in this work. Feedforward uses the measurement of an input disturbance to the plant as additional information for enhancing single loop PID performance. control This measurement provides an "early warning" that the controlled variable will be upset some time in the future. With this warning, the feedforward controller has the opportunity to adjust the manipulated variable before the controlled variable deviates from its set point. Note that the feedforward controller that does not use an output

of the process. This is the first example of a controller that does not use feedback control. Feedforward is usually combined with feedback so that the important features of feedback are retained in the overall strategy [2]. Feedforward control was not widely used in the process industries until the 1960s [3]. Since then, it has been applied to a wide variety of processes that include boilers, evaporators, solids dryers, direct fired heaters, and waste neutralization plants [4]. However, the basic concept is much older and was applied as early as 1925 in the three element level -control system for boiler drums.

## **Process Description**

The non-isothermal Continuous Stirred Tank Reactor (CSTR) where desired propylene glycol is produced by the hydrolysis propylene oxide is shown in Fig (1)



Fig.1, Continuous Stirred Tank Reactor

The feed stream consists of:

An equivalent mixture of propylene oxide and methanol.
 Water.

A cooling coil has been located for use in the hydration of propylene oxide; the reaction equation is:

$$CH_2 - CH - CH_3 + H_2O \xrightarrow{H_2SO_4} CH_2 - CH_2 - CH_3$$

The reaction is exothermic and takes place readily at room temperature when catalyzed by sulfuric acid [5]. The operating conditions used are given in Table (1).

Volumetric flow rate inlet	326.3 ft <sup>3</sup> /h (9.24m <sup>3</sup> /h)
Volume of liquid in reactor	$40.1 \text{ft}^3$ (1.136m <sup>3</sup> )
Temperature of feed inlet	535 R (297 K)
Temperature in reactor	564 R (313.35 K)
Coolant temperature	545 R (302.7 K)
Concentration of propylene oxide inlet	0.132 lbmol/ft <sup>3</sup> (2.11kmol/m <sup>3</sup> )
Concentration of propylene oxide in reactor	0.08316 lbmol/ft <sup>3</sup> (1.33 kmol/m <sup>3</sup> )

Table 1-A,	Operating	conditions	[11]
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Table 1-B, Properties [11]

Table 1-D, Properties [11]	
Ideal gas constant	1.987Btu/Ibmol.R
	(8.314
	kJ/kmol.K)
Heat of reaction	-36652Btu/Ibmol
	(85278.6
	kI/kmol)
Activation energy	32400Btu/Ibmol
	(75385.43
	kJ/kmol)
Overall heat transfer	100Btu/h.ft <sup>2</sup> .F
coefficient	$(1.36 \text{ kJ/h.m}^2.\text{K})$
Area for heat exchange	$40 \text{ ft}^2 (3.715 \text{ m}^2)$
Density* Heat capacity	52.857 Btu/ft <sup>3</sup> .F
	$(2.36 \text{ kJ/m}^3.\text{K})$
Pre exponential factor	$16.96*10^{12} (h^{-1})$
(ko)	

### **Mathmatical Model**

Mathematical models of chemical systems were developed for many reasons. Thus, they may be constructed to assist in the interpretation of experimental data for predicting the consequence of changes of system input or operating conditions, for deducing optimal system or operating conditions and for control purposes.

Usually there is an interest for dynamic model made to design and/or test the proposed control system. The dynamic and steady state simulation model for non isothermal (CSTR) consists of a system of equations based on mass and energy balances on the continuous stirred tank reactor (CSTR).

## 1. Overall Material Balance

$$v_{in}\rho_{in} - v_{out}\rho = \frac{dV\rho}{dt} \qquad \dots(1)$$

Assuming a constant amount of material in the reactor  $\frac{dV\rho}{dt}=0$ , we find that:

$$v_{in}\rho_{in} = v_{out}\rho$$

If we also assume that the density remains constant, then:

$$v_{in} = v_{out} = v$$
 and  $\frac{dV}{dt} = 0$ 

## 2. Balance on Component A

The balance on component A, assuming a constant volume reactor, is:

$$V \frac{dC}{dt} = v C_{Ai} - v C_{A} - rV \qquad \dots (2)$$

Where r is the rate of reaction per unit volume

#### **3. Energy Balance Around Tank**

The energy balance, assuming a constant volume, heat capacity and density, is:-

$$V\rho \quad C \quad \frac{dT}{p} = v \quad \rho \quad C \quad p \quad (T - T) + (-\Delta H)rV - UA(T - T) \quad \dots (3)$$

$$\Delta H(T) = \Delta H(T_{ref}) + \Delta C_p (T - T_{ref}) \qquad \dots (4)$$

Where  $(-\Delta H)r V$  is the rate of energy contributed by the exothermic reaction. The reaction rate per unit volume (Arrhenius expression) is:

$$r = k_o \exp(\frac{-\Delta E}{RT})C_A \qquad \dots (5)$$

Where the reaction is first order in propylene oxide concentration.

## Linearization of Dynamic Equations

The stability of the nonlinear equations can be determined by finding the following state space form:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

And determining the eigen values of the A (state space) matrix.

The nonlinear dynamic equations are:

$$f_1(C_A, T) = \frac{dC_A}{dt} = \frac{\upsilon}{V} C_{Ai} - \frac{\upsilon}{V} C_A - k_o \exp(\frac{-\Delta E}{RT}) C_A$$
....(6)
$$f_2(C_A, T) = \frac{dT}{dt} = \frac{\upsilon}{V} T_i - \frac{\upsilon}{V} T + (\frac{-\Delta H}{\rho C_p}) k_o \exp(\frac{-\Delta E}{RT}) C_A$$

$$- \frac{UA}{V\rho C_p} T + \frac{UA}{V\rho C_p} T_c$$
....(7)

The state and input variables can be defined in deviation variable form:

$$x = \begin{bmatrix} C_A - C_{As} \\ T - T_s \end{bmatrix}$$
$$u = \begin{bmatrix} v - vs \\ Tc - T_{cs} \end{bmatrix}$$

The state space A matrix is:

$$A = \begin{bmatrix} -\frac{\upsilon}{V} - k_{s} & -CA_{s}k'_{s} \\ \frac{(-\Delta H)}{\rho C_{P}}k_{s} & -\frac{\upsilon}{V} - \frac{UA}{V\rho C_{P}} + \frac{(-\Delta H)}{\rho C_{P}}CA_{s}k'_{s} \end{bmatrix}$$

Where

$$ks = k_o \exp(\frac{-\Delta E}{RTs})$$

$$k's = k_o \exp(\frac{-\Delta E}{RTs})(\frac{\Delta E}{RTs^2})$$

The state space B matrix is:

$$\mathbf{B} = \begin{bmatrix} \frac{\mathbf{C}\mathbf{A}_{i} - \mathbf{C}\mathbf{A}_{S}}{\mathbf{V}} & \mathbf{0} \\ \frac{\mathbf{T}i - \mathbf{T}s}{\mathbf{V}} & \frac{\mathbf{U}A}{\mathbf{V}\rho C_{P}} \end{bmatrix}$$

#### **Model Identification**

The IMC tuning is based on an assumed process model and leads to analytical expressions for the controller settings [6]. In this work, the process model is described by first order plus dead time (FOPDT) model. The FOPDT model parameters are found from the dynamic step response. For this FOPTD model, Sundaresanand Krishnaswamy's Method is used. This method avoids the use of the point of inflection construction entirely to estimate the time delay. They proposed that two times,  $(t_1)$  and  $(t_2)$  be estimated from a step response curve in Figs (3,4,5 and 6) corresponding to the 35.3% and 85.3% response times, respectively. The time delay (td) and time constant  $(\tau)$  are then estimated from the following equations:

$$td = 1.3 t_1 - 0.29 t_2 \dots (8)$$

$$\tau = 0.67 \ (t_2 - t_1) \qquad \qquad \dots (9)$$

These values of td and  $\tau$  approximately minimize the difference between the measured response and the model, based on a correlation for many data sets [6].

The transfer functions for step change in manipulated variables and disturbances are given by these equations:

$$Gp_1 = \frac{CA(s)}{\upsilon(s)} = \frac{0.0006324}{0.4251s + 1} e^{-0.0211 s}$$

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$$Gd_1 = \frac{CA(s)}{CAi(s)} = \frac{-2.057}{0.4605s + 1} e^{-0.3425s}$$

$$Gp_2 = \frac{T(s)}{Tc(s)} = \frac{1.565}{0.4644s + 1} e^{-0.0552 s}$$

$$Gd_2 = \frac{T(s)}{Ti(s)} = \frac{6.746}{0.46s+1} e^{-0.0591 s}$$

# Control Strategies

## 1. Internal Model Control (IMC)

A more comprehensive model-based design method, Internal Model Control (IMC), was developed by Garcia and Morari [7] and Rivera et al., [8]. The IMC method is based on the simplified block diagram as shown in Fig (2).



Fig.2, IMC control

A process model  $\check{G}$  and the controller output P are used to calculate the model response,  $\tilde{y}$ . The model response is subtracted from the actual response Y and the difference Y -  $\tilde{y}$  is used as the input signal to the IMC controller Gc<sup>\*</sup>. In general,  $Y \neq \tilde{y}$  due to modeling errors (G  $\neq$   $\check{G}$ ) and unknown disturbances  $D\neq$  0 that are not accounted for in the model.

The IMC controller is designed in two steps:

**Step 1.**The process model is factored as

Where  $\check{G}$ + contains any time delays and right-half plane zeros. In addition,  $\check{G}$ + is required to have a steady-state gain equal to one in order to ensure that the two factors in the equation are unique.

Step 2. The controller is specified as

$$\operatorname{Gc}^* = \frac{f}{\tilde{G}} - \dots(11)$$

where f is a low-pass filter with a steady-state gain of one. It typically has the form [8]:

$$f = \frac{1}{(\tau_C + 1)^n}$$

 $\tau_c$  is the desired closed-loop time constant. Parameter n is a positive integer. The usual choice is n= 1. Note that the IMC controller in Eq. (11) is based on the invertible part of the process model,  $\breve{G}$ -, rather than the entire process model,  $\breve{G}$  [6]. The choice of design parameter  $\tau_c$  is a key decision in IMC design method. Several IMC guidelines for  $\tau_c$  have been published for the FOPTD model. 1.  $\tau_c/td > 0.8$  and  $\tau_c > 0.1 \tau$  [8] 2.  $\tau > \tau_c > td$  [9] 3.  $\tau_c = td$  [10]

#### **2. Feedforward Control**

Feedforward control is a powerful strategy for control problems where important disturbance variable can be measured By measuring on-line. disturbances and taking corrective action before the controlled variable is upset, feedforward control can provide dramatic improvements for regulatory control. The chief disadvantage of feedforward control is that the disturbance variable must be measured (or estimated) on-line, which is not always possible.

Feedforward controllers tend to be custom-designed for specific applications, although a lead-lag unit is often used as a generic feedforward controller. The design of a feedforward controller requires knowledge of how the controlled variable responds to changes in the manipulated variable and the disturbance variable. This knowledge is usually represented as a process model. Steady-state models can be used for controller design; however, it may then be necessary to add a lead-lag unit to provide dynamic compensation. Feedforward controllers can also be designed by using dynamic models [6].

The design equation for feedforward control is:

$$G_{FF}(s) = \frac{Gd(s)}{Gp(s)} \qquad \dots (12)$$

This equation demonstrate that feedforward control depend heavily on a good knowledge of the process model (Gp, Gd). Perfect control necessitates perfect knowledge of Gp and Gd, which is not practically possible and this is considered the main drawback of feedforward control [11].

In practical applications, feedforward control is normally used in combination with feedback control. Feedforward control is used to reduce the effects of measurable disturbances, while feedback trim compensates for inaccuracies in the process model, measurement error, and unmeasured disturbances.

In a typical control configuration, the outputs of the feedforward and feedback controllers are added together, and the sum is sent as the signal to the final control element. Another useful configuration for feedforward-feedback control is to have the feedback controller output serve as the set point for the feedforward controller [6].

# **Results and Discussions**

The dynamic and model responses are studied for step change in the manipulated variables (v and Tc) and in the disturbance variables (CAi and Ti) in order to study these effect on the controlled variables (CA and T).

Figs. (3) and (4) show dynamic and model responses for the concentration of propylene oxide in reactor (CA) with time by step change in the volumetric flow rate for inlet (v) and the concentration of propylene oxide in feed stream (CAi), respectively.

In Figs. (3) and (4), it can be seen that the increase in the input flow rate is directly proportional to the concentration of propylene which led to a decrease in it.



Fig.3, Concentration versus time at step change in input flow rate



Fig.4, Concentration versus time at step change in input concentration

Figs. (5) and (6) show dynamic and model responses for temperature of reactor with time by step change in the temperature of coolant (Tc) and the temperature in feed stream (Ti), respectively.

From Figs. (5) and (6), it can be seen that the increase in the temperature of coolant is directly proportional to the temperature of the reactor; also the increase in the input temperature was found to lead to increase in it.

In this work, the unit step change is taken in the set point and disturbance of concentration and temperature within reactor using Feedback, Feedforward and Feedforward with Feedback controllers.



Fig.5, Temperature versus time at step change in coolant temperature



Fig.6, Temperature versus time at step change in input temperature

Figs (7) and (8) indicate the comparison among set point response of all used controllers for control on

concentration and temperature, respectively.

These figures show that the IMC controller provides an excellent set point response, while the Ziegler – Nichols controller provides the worst for the set point change because significant overshoots and longer settling times, and feedforward control provides sluggish for the set point change when reaching the steady state in a long time.



Fig.7, The comparison among set point response of all used controllers for control on concentration



Fig.8, The comparison among set point response of all used controllers for control on temperature

Table (2) shows that the value of ITAE for unit step change in set point of the IMC controller is less than the other controllers.

concentration within reactor (set point change)		
Controllers	ITAE	
PI controller	0.0105	
PID controller	0.0038	
IMC controller	0.0026	
Feedforward controller	0.0755	
Feedforward with feedback controller	0.0036	

Table 2-A, ITAE values for control ofconcentration within reactor (set point change)

Table 2-B, ITAE values for control oftemperature within reactor (set point change)

Controllers	ITAE
PI controller	0.052
PID controller	0.0166
IMC controller	0.0108
Feedforward controller	0.0747
Feedforward with feedback controller	0.0161

Figs (9,10,11and12) indicate to the comparison among disturbance response of all used controllers.

These figures show that the Ziegler – Nichols controller provides the best control for the disturbance, while the IMC controller produces an unacceptably slow disturbance response because the value of  $\tau_I$  is very large.

Feedforward controller provides a better response to disturbance than feedback controller with the remaining deviation (offset). Thus a combined feedforward-feedback control system is preferred in practice where deviation (offset) in feedforward controller has disappeared and feedforward controller is used to reduce the effects of disturbances, while measurable compensates for feedback trim inaccuracies in the process model, measurement error, and unmeasured disturbances.



Fig.9, The comparison among load response of PI, PID and IMC controllers for control on concentration



Fig.10, The comparison between load response of FF and FF-FB controllers for control on concentration



Fig.11, The comparison among load response of PI, PID and IMC controllers for control on temperature



Fig.12, The comparison between load response of FF and FF-FB controllers for control on temperature

Table (3) shows that the value of ITAE for unit step change in disturbance of the feedforward-feedback controller is less than the other controllers.

Table 3-A, ITAE values for control of concentration within reactor (disturbance change)

Controllers	ITAE
PI controller	0.0055
PID controller	0.0025
IMC controller	0.0717
Feedforward controller	0.0076
Feedforward with	0.0014
feedback controller	

Table 3-B, ITAE values for control of temperature within reactor (disturbance change)

Controllers	ITAE
PI controller	0.0428
PID controller	0.0250
IMC controller	0.4460
Feedforward controller	8.0943e004
Feedforward with	5.7434e004
feedback controller	

#### Conclusions

In this work, the mathematical model of the dynamic behavior of non isothermal process in a continuous stirred tank reactor (CSTR) was studied and developed.

The unit step change is taken in the set point and disturbance of concentration and temperature within the reactor using PI, PID, IMC, Feedforward and Feedforward with Feedback controllers.

A control system designed for process should provide fast and accurate changes for both the set point changes as well for a load changes.

The following conclusions can be drawn:

- 1. Feedback controller settings were calculated using the Ziegler-Nichols and IMC tuning and it was found that the Ziegler-Nichols controller provides the best control for the disturbance with a small maximum deviation and the worst for the set point change where it has significant overshoot, while the IMC controller produces satisfactory set-point responses and reaches the steady state in less time with lower overshoot, but an unacceptably slow disturbance responses because the approximate FOPTD model has relatively small time delay.
- 2. Feedforward feedforward and combined with feedback control systems were used as another strategy to be compared with the above strategy. Feedforward controller provides slow responses for the set point change when reaching the steady state in a long time but it produces excellent responses to disturbance change than feedback controller with the remaining deviation (offset). Thus, a combined feedforward-feedback control system is preferred in practice where deviation has disappeared.

In practical applications, feedforward control is normally used in combination with feedback control. Feedforward control is used to reduce the effects of measurable disturbances, while feedback trim compensates for inaccuracies in the process model, measurement error and unmeasured disturbances.

## Nomenclature

Nomer	nclature
Kc	Proportional gain
(pressu	re/error)
τI	0
τd	Derivative time (hr)
А	Area for heat exchange (m2)
CA	Concentration of propylene
	n reactor (kmol/m3)
	Concentration of propylene
oxide	in reactor at steady state
(kmol/i	
	Concentration of propylene
oxide i	n feed stream( kmol/m3)
Ср	1
υ	Volumetric flow rate for inlet
(m3/hr	)
υs	Volumetric flow rate at steady
state (	
ko	pre exponential factor (hr-1)
R	Ideal gas constant (kJ/kmol*K)
r	Rate of reaction per unit
volume	e (kmol/m3*hr)
t	Time (hr)
Tref	Temperature of reference (K)
Tc	Temperature of coolant (K)
Tcs	Temperature of coolant at
•	state (K)
Ti	Temperature of feed inlet ( K )
Т	Temperature of reactor ( K )
Ts	Temperature of reactor at
•	state (K)
V	Volume (m3)
U	Overall heat transfer coefficient
	(kJ/hr*m2* K)
ΔE	Activation energy (kJ/kmol)
(-ΔH)	Heat of reaction (kJ/kmol)
ρ	Density (kg/m3)
td	Time delay (hr)
Т	Time constant (hr)
ЪŢ	Desired closed loop time
constar	nt (hr)

# Abbreviation

ADDreviation				
CSTR Continues stirred tank reactor				
FOPD	T First	order	plus	dead
time				
PI	Proportiona	l integra	l contr	oller
PID	Proportional integral derivative			
IMC	Internal mod			
Ğ Ğ+	Process mod	lel		
Ğ+	Non invertib	ole part o	of the	
	process mod	lel		
Ğ-	Invertible pa	art of the	e proce	ess
model				
Gc*	Transfer f	unction	for	IMC
contro	ller			
Р	Controller output			
ỹ Y	Model response			
Y	Actual respo	onse		
f	Low pass fil	ter		
Gp	Gp Transfer function for process			
Gd	Transfer	functi	ion	for
disturbance				
GFF	Transfer	functi	ion	for
feedforward				
FF	Feedforward controller			
FF-FB Feedforward - Feedback				
controller				
ITAE	Integral time	e absolu	te erro	r
-				

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