

Drag Forces under Longitudinal Interaction of Two Particles

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Abstract

Direct measurements of drag force on two interacting particles arranged in the longitudinal direction for particle Reynolds numbers varying from 10 to 10^3 are conducted using a micro-force measurement system. The effect of the interparticle distance and Reynolds number on the drag forces is examined. An empirical equation is obtained to describe the effect of the interparticle distance (l/d) on the dimensionless drag.

Keywords: Hydrodynamic interaction, drag coefficient.

Introduction

The knowledge of the particle drag force is important in the flow analysis of particulate and multiphase flow systems.

For an isolated particle or for particles in dilute system, the particle drag force can be obtained theoretically or semi-empirically. However, in concentrated particulate systems, particle interaction become important and thus the particle drag force in these systems may deviate significantly from that of an isolated particle.

Theoretical work on the drag force of interacting particles is limited to very low Reynolds number, due to the nonlinearity of the equation governing the flow motion at higher Reynolds number. The only multiparticle system from which the drag force can be rigorously determined analytically is a two-particle system. Stimson and Jeffery [1] employed a bipolar coordinate system to solve the velocity field of a slow-moving fluid flowing around two equal-sized particles aligned in the flow direction. The major research efforts on the drag force of individual interacting particles have mainly focused on unbounded Stokes flows Happel and Brenner [2]. They reported that for all the centerline orientations the particle drag force is lower than that of a single particle.

Outside the Stokes flow regime, there have been only a few fundamental studies dealing with the drag force of a small number of particles.

Lee and Tusji et al [3] conducted experiments on the interactions between two particles at Reynolds number about 10^4 and Reynolds number from 10^2 to 10^3 , respectively. Their results showed that the drag force of the trailing particle decreased with decreasing distance between particles, but the particle drag force increased as the other particle approached from the transverse direction.

For lower Reynolds number, Rowe and Henwood [4] presented a diagram of the drag ratio versus angular displacement at three different separation distances for $Re = 96$. Zhu et al [5] used a micro balance to measure the drag force on two interacting particles arranged in the longitudinal direction for Re from 20 to 130 . They reported a similar trend in the drag ratio versus separation distance for the trailing particle.

Yuan and Prosperetti [6] evaluated the "true drag coefficient" of a pair of bubbles rising in line in the intermediate Reynolds numbers range ($50 \leq Re \leq 200$). They showed that the drag coefficient of the leading bubble was almost the same as the one of the single bubble; however the one of the trailing bubble greatly decreased due to the flow field generated by the leading bubble.

Liang et al [7] experimentally measured the forces in a number of particle orientations. The main findings of their research showed that the drag force experienced by a particle is significantly affected by surrounding particles, more so when the particles are co-aligned.

Doods and Naser [8] studied three model configurations to examine the individual effect of particle arrangement on the experienced drag of a particle for Reynolds number from 1 to 50. For an infinite number of co-aligned stream-wise particles, their results showed that a significant reduction in drag force was experienced by the particle, compared to that of an isolated particle, especially at small separation distances and higher Reynolds numbers.

Hollander and Zaripov [9] used the theoretical values of the drag reduction according to the results for the middle sphere in chains of 7 to 101 spheres at different spacing distances for Reynolds number from 1 to 10.

For Reynolds number from 10 to 10^3 , no reliable quantitative results of the drag force of a particle under the influence of other particles are available, thus it need to be explored.

In this study, a microforce measuring system is developed to directly measure the drag force between two co-aligned particles for Reynolds number varying from 10 to 10^3 which widely observed in the particle applications. The drag force is represented by an empirical equation based on the present measurements.

Experimental Work

The experiments were performed in a QVF cylindrical column of length 2.0 m and a diameter of 0.3 m (as shown Fig. 1).

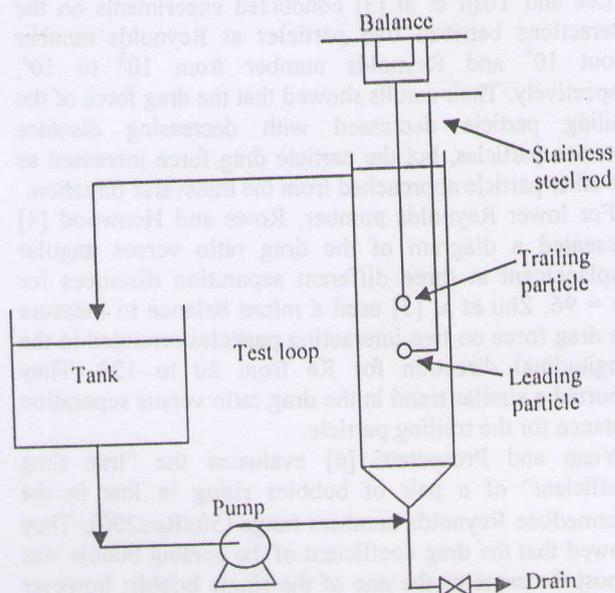


Fig. 1 Schematic diagram of experimental apparatus

A glycerin /water solution of approximately 80 wt % ($\rho_f = 1208.5 \text{ kg/m}^3$, $\mu_f = 60.1 \text{ mPa.s}$), is circulated upwards through column. With this viscous fluid, the drag force acting on the particle can be increased to the accurately detectable range of the micro-balance, which has the resolution of 1 mg.

The fluid viscosity was measured by a Fann V-G meter (Baroid) and density was measured by a standard hydrometer. Stainless steel spheres ($\rho_s = 7700 \text{ kg/m}^3$) of 15.4 mm diameter were used as test particles. By changing the fluid velocity, Reynolds number varies from 10 to 10^3 .

The test particle is attached to an electronic balance (Sartorius BL 210S) through a thin rod (about 1.6 mm diameter) such that the drag acting on the test particle can be measured.

Drag force measurements were conducted on the test particle attached to the rod and on the rod only. The particle drag force can be obtained by subtracting the drag force of the rod from that of the test particle with the rod. The total drag on the test particle and its supporting rods was obtained by deducting the gravitational and buoyancy contributions from the total force measured by the balance.

The gravitational and buoyancy forces are pre-determined in the stationary fluid and the drag coefficient of the particle can be calculated from:

$$F_{Do} = C_{Do} \frac{\pi}{8} \frac{\mu_f^2}{\rho_f} Re^2 \quad (1)$$

Results and Discussion

It was found that in all the experimental conditions, the drag force of the supporting rod contributes about 30% of the total measured drag force on the particle-rod system.

The drag force measured system is validated by comparing the drag coefficient of a single particle with that reported in the literature, Schlichting [10] as shown in Fig. 2, the agreement is reasonable.

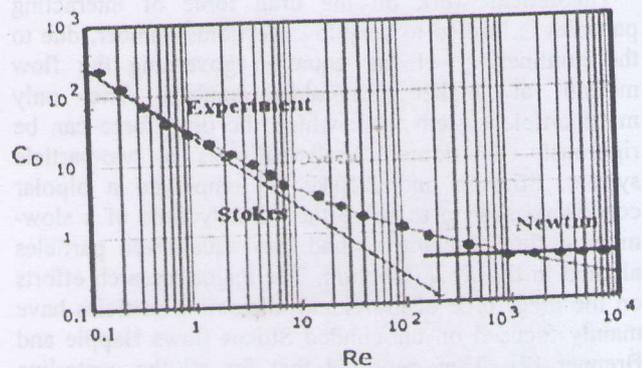


Fig. 2 Predictions of various models for the drag coefficient for a spherical particle

Typical results of the drag of the trailing particle under the influence of the leading particle are illustrated in Fig. 3. The drag is non-dimensionalized by the drag force of a single non-interacting particle under the same Reynolds number. The interactive distance (l) is also expressed in a dimensionless form l/d .

Fig. 3 shows that the drag ratio of the trailing particle decreases exponentially with decreasing l/d and reaches a minimum at the contact position, which agrees with the results of Rowe and Henwood [4] and Tsuji et al [11]. The reduction in the drag ratio is caused by the wake effect.

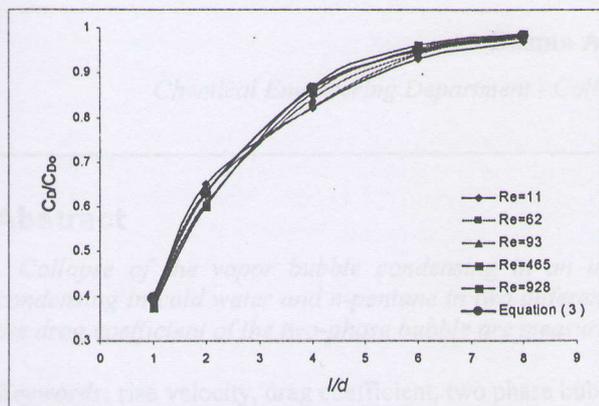


Fig. 3 Experimental data for the variation in the drag ratio with inter-particle distance

An empirical relation is obtained to describe the effect of the interparticle distance l/d on the dimensionless drag of the trailing particle. The empirical equation takes the exponential form [12]:

$$\frac{F_D}{F_{D0}} = \frac{C_D}{C_{D0}} = 1 - A \exp\left(-B \frac{l}{d}\right) \quad (2)$$

Where F_D is the drag force of an interactive particle and F_{D0} is the drag force of a single non-interacting particle. The coefficients are determined as, $A=1.0$ and $B=0.5$, thus equation 2 can be approximately written as:

$$\frac{F_D}{F_{D0}} = \frac{C_D}{C_{D0}} = 1 - \exp\left(-0.5 \frac{l}{d}\right) \quad (3)$$

Equation 3 is examined at two limits of the separation distances between two spheres. As l goes to infinity, the second term in the equation vanishes and the drag ratio becomes unity as expected. At contact ($l/d=1$), the drag ratio equals about 0.4, which gives the minimum value of the drag ratio.

The drag is affected by the interaction, it decreases with decreasing distance between the spheres, but the effect of interaction disappears at a distance larger than

$l/d = 5$ to 10 and asymptotically approaches the single sphere value.

The reason for the drag ratio reduction with decreasing inter-particle according to Zhu et al. [5] is that the particle interaction renders the wake vortex of the leading particle longer than that of a single non-interacting particle.

It is also found that the curves for different Reynolds number may cross each other at l/d of about 1 to 3. At a small l/d , the drag ratio of a higher Reynolds number decrease faster within the Reynolds number range tested.

Conclusions

1. An empirical relation is obtained in this investigation to describe the drag force variation of a single particle trailing in the wake of a leading particle.
2. The drag ratio of the trailing particle decreases exponentially with decreasing l/d and reaches a minimum at the contact position, but the effect of interaction disappears at a distance larger than l/d of about 5 to 10 and asymptotically approaches the single sphere value.
3. It is found that the curves for different Reynolds number may cross each other at l/d of about 1 to 3.

Nomenclature

C_D	drag coefficient (—)
C_{D0}	drag coefficient of an isolated sphere (—)
d	sphere diameter (m)
F_D	drag force (N)
F_{D0}	drag force of an isolated sphere (N)
g	gravitational acceleration (m/s ²)
l	the distance between the centers of spheres (m)
Re	Reynolds number based on the sphere diameter (ud/v) (—)
u	sphere velocity (m/s)

Greek letters

μ_f	dynamic viscosity of fluid (kg/m.s)
ρ_f	density of fluid (kg/m ³)

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