SIMULATION OF RADIAL, REAL GAS FLOW

Hamad-Allah S. M.
Petroleum Engineering Department – College of Engineering – University of Baghdad – Iraq

ABSTRACT

A numerical single phase, one-dimensional model has been developed to simulate the flow of gas towards a well. The principal direction of flow is radial. The partial differential equation that describes the flow of gases is used in term of real gas potential (ψ). This equation is converted to a finite difference form. The drainage area is divided into nonuniform blocks by using logarithmic discretisation. The only unknown in the flow equation is ψ at each grid point. The set of algebraic equations is solved simultaneously by a direct method. The pressure at each grid is calculated from ψ by using linear interpolation.

The simulator is used to calculate the bottom hole pressure in a draw down test. It has been found that tuning skin factor and permeability yields the best match in the draw down test example used.

INTRODUCTION

Natural gas production has become increasingly important and now provides about one-fifth of the world primary energy requirement (Ikoku 1980). In accordance, researches are devoting their effort to emphasize on gas reservoir engineering problems. One of the most important subjects of gas reservoir engineering is flow of real gas in the reservoir. Several analytical solutions, which concern gas flow, are presented in the literatures.

In pressure transient test, pressure change is measured as a function of time. Important formation properties of potential value in optimizing either individual completion or depletion plane for a reservoir can be determined from pressure transient testing. Gas flow in a reservoir is influenced by damage or improvement to the permeability near the wellbore and by the high velocity flow phenomena.

The pressure changes with time can be also calculated by numerical simulation. In addition, skin factor, permeability and other reservoir descriptions can be estimated. Simulation is a powerful method available for solving the difficult problems. The difficulty arises due to heterogeneity, irregular shape of the reservoir, and nonlinearity of some equations. For such cases there is no analytical solutions.

In 1970, Kazemi presented a numerical solution that describes pressure behavior in a bounded commingled reservoir. Lee and Wattenbarger (1996) constructed a two dimensional reservoir simulator for real gas flow, which can be used with r-z and x-y geometry. Allawy (1999) studied the pressure behavior in a multilayered gas reservoir with and without crossflow. He used cylindrical coordinate in his model.

In the present study, a simulator has been constructed to model single phase, real gas flow in single dimension. Many reservoir parameters have been tested in order to sieve these that mainly affect history matching.

Analytical Solution of Real Gas Flow in Porous Media

To develop an equation that describes the flow of fluid in porous media, several simplifying assumptions about the well and reservoir must be made. These assumptions are used when needed with the principle of mass conservation, Darcy's law, and equation of state (EOS) which are combined to derive the diffusivity equation. The following is the diffusivity in radial coordinate system that can be used to model gas flow in the reservoir:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{P}{\mu(P)Z(P)}r\frac{\partial P}{\partial r}\right]=\frac{\varphi}{K}\frac{\partial}{\partial t}\left[\frac{P}{Z(P)}\right] \quad (1)$$

In the above equation, it is assumed that the medium is homogeneous, the flowing gas is of constant composition, and the flow is laminar and isothermal.

Many forms of diffusivity equation can be derived according to the treatment of the pressure dependent variables. If $P/\mu Z$ is assumed constant, Eq.(1) will be transformed to the following form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial P}{\partial r}\right]=\frac{\varphi\mu C_g}{0.006328K}\frac{\partial P}{\partial t} \quad (2)$$

If it is assumed that viscosity and Z-factor change slowly with change in pressure, the following form results:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial (P)^2}{\partial r}\right]=\frac{\varphi\mu C_g}{0.006328K}\frac{\partial (P)^2}{\partial t} \quad (3)$$

To limit the assumptions, Al-Hussainy Ramey and Crawford proposed using the real gas pseudo pressure or real gas potential m(p):

$$m(p)=\psi=2\int_{P_o}^{P}\frac{P}{\mu(P)Z(P)}dP \quad (4)$$

where Po is a low base pressure.

Using Al-Hussainy (1966) approach, the diffusivity equation will be in the following form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial \psi}{\partial r}\right]=\frac{\varphi\mu C_g}{0.0002637K}\frac{\partial \psi}{\partial t} \quad (5)$$

The diffusivity equation in any form can be used to develop analysis and design technique for transient testing.

Numerical Solution of Real Gas Flow

In this work, the radial model is used in simulating gas flow. This model is one-dimensional in which the flow is assumed in r-direction. In other words, the flow in z-direction is neglected.

The equation of gas flow in terms of ψ or m (p) is chosen and approximated by using Taylor series. For a heterogeneous reservoir, the diffusivity equation can be written as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[0.006328Kr\frac{\partial \psi}{\partial r}\right]=\varphi\mu C_g\frac{\partial \psi}{\partial t} \quad (6)$$

It seems convenient to use the approach of Bruce, Peaceman, and Rachford (1952) in which they converted the above equation from a radial form to nearly linear form by the following transformation:

$$u=\ln\frac{r}{r_w} \quad (7)$$

So that

$$\frac{du}{dr}=\frac{1}{r} \quad (8)$$

substituting Eqs. (7) & (8) in Eq.(6) results in:

$$\frac{1}{r^2}\frac{\partial}{\partial u}\left[0.006328Kr\frac{\partial \psi}{\partial u}\right]=\varphi\mu C_g\frac{\partial \psi}{\partial t} \quad (9)$$

The finite difference approximation of Eq.(9) is:

$$\frac{0.006328}{r_i^2}\left[\frac{\left(K\frac{\partial\psi}{\partial u}\right)_{i+1/2}-\left(K\frac{\partial\psi}{\partial u}\right)_{i-1/2}}{\Delta u}\right]=\varphi_i\mu_i C_{g_i}\left(\frac{\psi_i^{n+1}-\psi_i^n}{\Delta t}\right) \quad (10)$$

In Eq.(10), central difference is used for approximating a spatial derivative. The superscript n and n+1 indicate the old and new time level respectively.

Grid Construction

In this study, the method of Similote (1974) is used in constructing the grids. Similote adopted a block-centered scheme and he used the well bore radius and drainage radius in discretising the simulated area. The following definitions are used to construct the blocks:

$$\alpha = \frac{u}{I} \tag{11}$$

where $$u = \ln \frac{r_e}{r_w} \tag{12}$$

It is obvious from Eqs. (11)& (12) that α is constant. The block boundary radii are defined as:

$$r_{i+1/2} = r_w e^{i\alpha} \tag{13}$$

and the block-centered radii are defined as:

$$r_i = r_w e^{((i-1/2)\alpha)} \tag{14}$$

Using the above definitions and adopting the backward difference for approximating the time derivative, Eq.(10) becomes:

$$\frac{0.006328}{r_i^2 \alpha^2}\left[K_{i+1/2}\psi_{i+1}^{n+1} - \left(K_{i+1/2} + K_{i-1/2}\right)\psi_i^{n+1} + K_{i-1/2}\psi_{i-1}^{n+1}\right] = \varphi_i \mu_i C_{gi}\left(\frac{\psi_i^{n+1} - \psi_i^n}{\Delta t}\right) \tag{15}$$

Equation (15) is called an implicit finite difference equation because it involves more than one unknown. There are three unknowns ψ_{i+1}^{n+1}, $\psi_i^{n+1}, \psi_{i-1}^{n+1}$.

Multiplying each sides of Eq. (15) by $\pi h\left(r_{i+1/2}^2 - r_{i-1/2}^2\right)$ yields:

$$\frac{0.006328\pi h_i\left(r_{i+1/2}^2 - r_{i-1/2}^2\right)}{r_i^2 \alpha^2}\left[K_{i+1/2}\psi_{i+1}^{n+1} - (K_{i+1/2} + K_{i-1/2})\psi_i^{n+1} + K_{i-1/2}\psi_{i-1}^{n+1}\right] = V_{pi}\mu C_{gi}\left(\frac{\psi_i^{n+1} - \psi_i^n}{\Delta t}\right) \tag{16}$$

where V_{pi} is the pore volume:

$$V_{pi} = \pi h_i \varphi_i (r_{i+1/2}^2 - r_{i-1/2}^2) \tag{17}$$

In order to convert the units of terms in Eq. (16) from CF/day to MSCF/day, it must be multiplied by the factor $\frac{T_{sc}}{1000 P_{sc} T}$

Now Eqs.(13) & (14) are used to make the following substitution:

$$\frac{r_{i+1/2}^2 - r_{i-1/2}^2}{r_i^2} = \frac{r_w^2 e^{2i\alpha} - r_w^2 e^{2(i-1)\alpha}}{r_w^2 e^{2(i-1/2)\alpha}} = e^{\alpha} - e^{-\alpha} \tag{18}$$

Let $E = 0.006328 \frac{T_{sc}}{1000 P_{sc} T} \frac{\pi(e^{\alpha} - e^{-\alpha})}{\alpha^2}$

After substituting Eq. (18) in Eq. (16) and with some manipulation it becomes:

$$Eh_i K_{i+1/2}\psi_{i+1}^{n+1} - Eh_i(K_{i+1/2} + K_{i-1/2})\psi_i^{n+1} + Eh_i\psi_{i-1}^{n+1} = \frac{T_{sc}}{1000P_{sc}T} V_{pi}\mu_i C_{gi}(\psi_i^{n+1} - \psi_i^n) + Q_i \tag{19}$$

Let
$$B_i = Eh_i K_{i+1} \tag{20a}$$
$$C_i = Eh_i K_{i-1} \tag{20b}$$

$$A_i = -B_i - C_i - \frac{T_{sc}}{1000 P_{sc} T}\frac{V_{pi}\mu_i C_{gi}}{\Delta t} \tag{20c}$$

$$D_i = -\frac{T_{sc}}{1000 P_{sc} T}\frac{V_{pi}\mu_i C_{gi}}{\Delta t}\psi_i^n + Q_i \tag{20d}$$

Hence Eq.(19) can be written as:

$$B_i \psi_{i+1}^{n+1} + A_i \psi_i^{n+1} + C_i \psi_{i-1}^{n+1} = D_i \tag{21}$$

Equation (21) is related to each of the grids in the simulated domain, i.e., for N grid points, there is N equation. The only unknown in this set of equations is the real gas potential of each grid at a new time level (n+1). The resulting N equations are solved simultaneously. A direct method, Thomas algorithm, is used to obtain ψ at each grid point. Linear interpolation is then used to obtain the pressure from ψ.

The matrix form of Eq. (21) can be written as:

$$A\vec{P} = \vec{d} \tag{22}$$

where A is a coefficients matrix and P and d are the column vectors. The matrix equations can be written as:

$$\begin{bmatrix} a_1 & c_1 & & & \\ b_2 & a_2 & c_2 & & \\ & b_3 & a_3 & c_3 & \\ & & & & \\ & & & b_N & a_N \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \\ P_N \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \\ d_N \end{bmatrix} \quad (23)$$

Notice that the non-zero elements follow a diagonal trend. This is called a tridiagonal matrix.

Test Example

This example is taken from Ikoku (1980). The following are the reservoir and gas data:

Table (1): Reservoir Properties

Pi= 2300 Psia	h= 10 ft	r_w = 0.5 ft	r_e = 2980 ft
T=130 °F	S_g = 0.77	ϕ=0.1	

Table (2): Gas Properties

Pressure, Psia	Z	Viscosity, cp
400	0.95	0.0117
800	0.90	0.0125
1200	0.86	0.0132
1600	0.81	0.0146
2000	0.80	0.0163
2400	0.81	0.0180

Table (3): Draw Down Data

Flowing time Hrs	Pwf, Psia Flow No.1 (Q=1600Mscf/d)	Pwf, Psia Flow No.1 (Q=3200Mscf/d)
0.232	1855	1105
0.4	1836	1020
0.6	1814	954
0.8	1806	906
1.0	1797	860
2.0	1758	700
4.0	1723	539
6.0	1703	387

This problem had been solved analytically to obtain average permeability K, skin factor S, and non-Darcy flow coefficient D. The last parameter is used in Darcy's law to account for high velocity gas flow in the porous media such as near the wellbore.

When solving this problem numerically the value of permeability calculated by the analytical solution is used in the simulator since there is no other estimation available for this property. This value is (4.5) md.

The drainage area is divided into (10) blocks. The simulator contains a subroutine for calculating ψ for each grid point using numerical integration. The pressure at initial condition (t=0) is (2300) Psia as given in Table (1). The following equation is used to calculate bottom hole pressure, P_{wf}

$$P_{wf} = \left[P_1^2 - \frac{1422 Q \mu_1 Z_1 T}{K_1 h} \left(\ln\frac{r_1}{r_w} - \frac{1}{2} + \bar{S}\right)\right]^{0.5} \quad (24)$$

and $$\bar{S} = S + DQ \quad (25)$$

where P_{wf} : Flowing bottom hole pressure.

P_1 : Pressure of the first grid point.

μ_1 : Viscosity at the pressure of the first grid point.

Z_1 : Deviation factor at the pressure of the first grid point.

K_1 : Permeability of the first grid point.

r_1 : Radius of the first grid point.

$\bar{S}$: total or apparent skin factor.

S : skin factor.

Note that in Eq.(24), the high velocity flow is included as additional skin factor resulting in an additional pressure drop.

Many runs have been performed using the current model in order to obtain a good match. The permeability and porosity are assumed constant in some runs and variable in others. Some times only the first grid properties are changed since they may be greatly affected by well operations. The apparent skin factor is also tuned-up. It has been found that neither changing porosity nor changing the permeability of the first grid point has significant effect on history matching. The main factors affected matching are the permeability of the whole grid points and the apparent skin factor. This conclusion is in agreement with the results obtained by Aziz and Settari (1979). Many groups of these parameters are tested and their values for the final match is given in Table (4). Note that the skin factor and non-Darcy flow coefficient are obtained by solving Eq.(25) simultaneously using the two flow rates. The results of the analytical solution are also given in this table.

Acceptable agreement is noticed between the results of the numerical model and these of the analytical solutions. Figure (1) and (2) shows the good match between the calculated and measured flowing bottom hole pressure, which has been obtained for the two drawdown tests. The pressure distributions at different production times are calculated by the present simulator and given in Fig.(3) and Fig.(4) respectively.

Table (4): Comparison between analytical and simulator results

Analytical solution			Numerical solution		
Kmd	S	D d/Mscf	K md	S	D d/Mscf
* 4.5 **4.9	-0.284 0.008	0.000513 0.000588	2.25	.47	.0001125

* ψ method ** P2 method

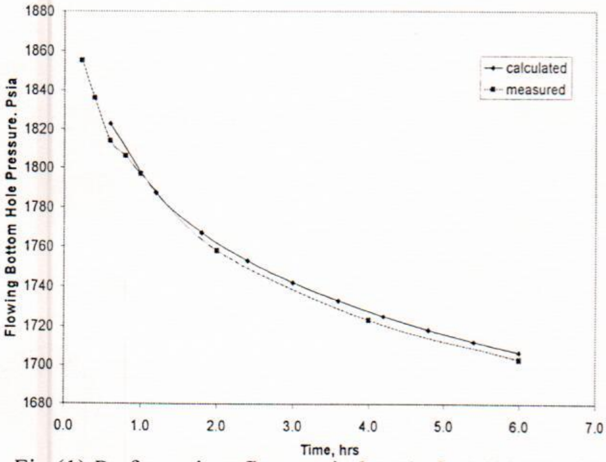

Fig.(1) Pwf vs. time flow period no.1, Q=1600 Mscf/d

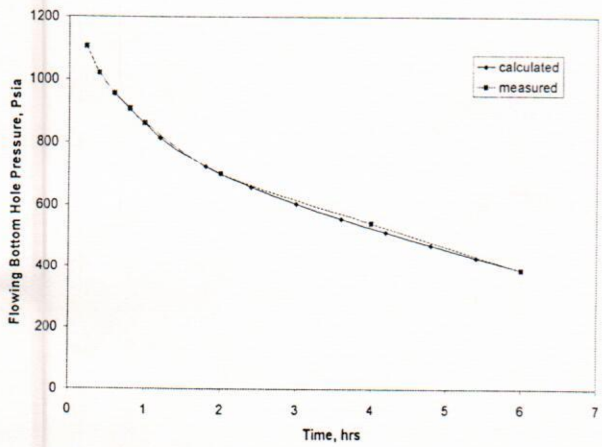

Fig.(2) Pwf vs. time flow period no. 2, Q=3200 Mscf/day

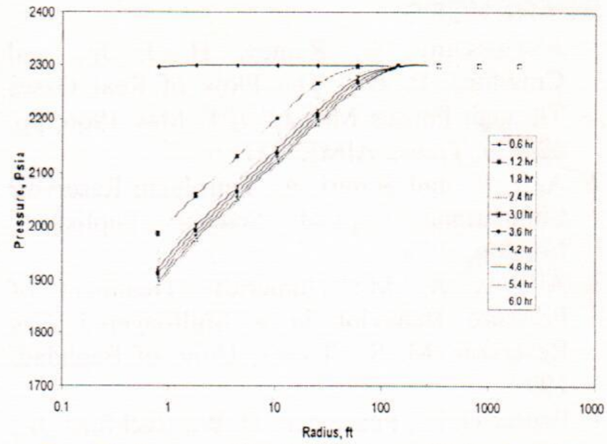

Fig. (3) Pressure Distribution in the Well, Q=1600 Mscf/d

CONCLUSION

Some well testing parameters of gas reservoir can be estimated by using the present model. The model is a single-phase one-dimensional in which an implicit scheme is adopted for solving the set of equations. The best match with the actual reservoir performance can be obtained by adjusting absolute permeability and apparent skin factor.

NOMENCLATURE

C_g	Gas compressibility, Lt2 /m, psi-1
h	Thickness, L, ft
K	Effective formation permeability, L2, md
m(p)	Gas pseudopressure, m/Lt^3, $psia^2/cp$
P	Pressure, m/Lt^2, psi
P_{sc}	Pressure at standard condition, m/Lt2, psi
P_{wf}	Flowing bottom hole pressure, m/Lt2, psi
Q	Flow rate, L3/t, Mscf/day
r	Distance from wellbore center, L, ft
r_w	Wellbore radius, L, ft
S	Skin factor, dimensionless
$\bar{S}$	Total or apparent skin factor dimensionless
t	Time, t, day
T	Reservoir temperature, T, °R
T_{sc}	Temperature at standard condition, T, °R
V_p	Pore volume, L3, ft3
Z	Gas law deviation factor, dimensionless

Greek symbols

μ	Viscosity, m/Lt, cp
φ	Porosity, fraction
ψ	Gas pseudopressure, m/Lt3, psia2/cp

Subscript & superscript symbols

i	grid block index
$i \pm 1/2$	boundary indices of grid block
n	time level

REFRENCES

1. Al-Hussainy, R, Ramey, H. J. Jr., and Crawford, P. B.:" The Flow of Real Gases Through Porous Media," JPT, May 1966, pp. 624-36; Trans., AIME, 237.
2. Aziz, K. and Settari, A.: Petroleum Reservoir Simulation, Applied Science Puplishers, London, 1979.
3. Allawy, A. M.: Numerical Treatment of Pressure Behavior in a Multilayered Gas Reservoir, M. Sc. Thesis, Univ. of Baghdad, 1999.
4. Bruce, G. H., Peaceman, D. W., Rachford, Jr., H. H., and Rice, J. D.:" Calculations of Unsteady State Gas Flow Through Porous Media," SPE Paper 3518, The Petroleum Branch Fall Meeting Houston, Tex., Oct. 1952.
5. Lee, J., Wattenbarger, R. A.: Gas Reservoir Engineering, SPE, Richardson, TX 1996.
6. Ikoku, C. U.: Natural Gas Engineering, PennWell Publishing Company, 1980.
7. Kazemi, H.:" Pressure Buildup in Reservoir Limit Testing of stratified Systems," JPT, Aug. 1966, pp. 997-1000.
8. Similote, V.: Mathematical Simulation of Gas and Water Coning in Petroleum Reservoirs, Ph. D. Dissertation, Univ. of Missouri-Rolla, 1974.