THEORETICAL ANALYSIS OF CONCENTRIC FLOW OF SPHERICAL CAPSULE IN LAMINAR FLOW

Farhan L. Rasheed, Ahmed A. Ghulfus, and Hameed B. Mahood
Department of Heat Transfer – Iraqi Atomic Energy Commission - Iraq

ABSTRACT

Predictions of capsule velocity for an idealized system in which a single spherical capsule moves concentrically with respect to the pipe in laminar flow condition.

Results were obtained for the transport by water of a single spherical capsule with density equal to that of the water in a 1.5 inch pipe. The diameter ratios were 0.39, 0.69 and 0.89. The average flow velocity varied from 0.11 to 0.23 ft/s (Re: 1000 to 2000).

The increase in average flow velocity leads to linear increase in capsule velocity, the reduction of the diameter ratio leads to a general increase in capsule velocity. good agreement in the behavior between the theoretical results (present work)and the experimental results obtained by Ellis and Bolt.

INTRODUCTION

Capsule pipelining is a concept of economical long distance solids transport whereby the solids are introduced into the pipeline in the form of a long train of cylinders or spheres with diameters approaching that of the pipe. The capsules, which may be rigid slugs, flexible bodies of coherent paste, or packaged commodities, are carried along by the flowing liquid[1]. Ellis[2] have reported fundamental work on the flow of single spherical capsules and single cylindrical capsules of various lengths and with square and ellipsoidal ends in a 1.25 inch plastic pipe. For both the spheres and the cylinders, diameter ratios were varied from 0.39 to 0.89 were studied. With the smallest spheres the flow pattern was extremely unstable and the spheres bounced from wall to wall, often spinning as they progressed down the pipe.

Ellis [3] showed that the percentage of sliding experienced by the spheres of both material (plastic and Aluminum) increased both with increase of water velocity and decrease of sphere diameter, and that it was considerably greater for the plastic spheres.

Liu et al. [4] developed a four-regime theory to predict the behavior of capsule or coal-log flow in pipes, including its pressure gradient, capsule velocity, capsule drag and lift.

In the present paper the pipeline flow of capsules will be considered from a theoretical point of view. The basic model assumes a spherical capsule aligned axially within the pipe in a concentric position. The present model therefore, certainly represents the flow of capsules having the same density as the fluid under laminar flow condition.

ANALYSIS

Consider a spherical capsule flowing in a concentric position relative to the pipe wall under the influence of a fluid stream in laminar flow. Under steady state conditions the fluid will be in un:directional motion and the basic differential equation (written in spherical co-ordinates) is (Whitaker):

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} = \frac{1}{\mu}\left(\frac{\Delta p}{L}\right)_c \qquad (1)$$

Where u is the velocity at radius r, μ is the viscosity of the fluid and (Δp/L)c is the constant pressure gradient (the subscript indicates that the pressure gradient applies to the capsule system). This equation may be rearranged to

$$\frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) = \frac{1}{\mu}\left(\frac{\Delta p}{L}\right)_c \qquad (2)$$

and integrated twice to give the fluid velocity. Application of the boundary conditions u=0 when r=ro and u=uc when r=rc. Where ro is the radius of the pipe, uc the capsule velocity and rc is the radius of the capsule yields:

$$u = \frac{1}{6\mu}\left(\frac{\Delta p}{L}\right)_c \left(r_o^2 - r^2\right) for\ r_c < r < r_o \quad (4)$$

Further,

u=uc for 0<r<rc (5)

The capsule velocity, uc is given by

$$u_c = \frac{-1}{6\mu}\left(\frac{\Delta p}{L}\right)_c \left(r_o^2 - r_c^2\right) \quad (6)$$

Equations 4,5 and 6 completely define the velocity profile. Now the volumetric flow rate of fluid in the annulus between the capsule and the pipe wall is given by

$$Q_{annulus} = 2\pi \int_{r_o - r_c \sin\theta}^{r_o} \int_{\theta=0}^{\pi} r.udr \quad (7)$$

Integration after substituting for u from equation (4) gives

$$Q_{annulus} = \frac{-\pi}{3\mu}\left(\frac{\Delta p}{L}\right)_c \left[\frac{\pi r_o^2 r_c^2}{2} - 2r_o r_c^3 + \frac{3\pi}{32} r_c^4\right] \quad (8)$$

$$= \frac{2\pi u_c}{\left(r_o^2 - r_c^2\right)}\left[\frac{\pi r_o^2 r_c^2}{2} - 2r_o r_c^3 + \frac{3\pi}{32} r_c^4\right] \quad (9)$$

The volumetric throughput of capsule is simply

$$Q_{capsule} = \frac{2\pi}{3} r_c^2 u_c \quad (10)$$

The average flow velocity is obtained by dividing the total volumetric flow rate by the cross-sectional area. Thus

$$u_{av} = \frac{Q_{annulus} + Q_{capsule}}{\pi r_o^2} \quad (11)$$

and using equations 9 to 10 we obtain

$$u_c = \frac{(1-k^2)u_{av}}{\pi k^2 - 4k^3 + \frac{3\pi}{16}k^4 + \frac{2k^2}{3}\left(1-k^2\right)} \quad (12)$$

In which k= rc/ro = d/D. Where d is the capsule diameter and D is the pipe diameter. Equation (12) represents the relationship between the capsule velocity and the average velocity under laminar conditions.

Later in this discussion we shall have reason to introduce the Reynolds number defined as ρ uav D/μ for the pipeline flow of a pure fluid where ρ is the density of the fluid, uav is the average velocity and D is the pipe diameter. In the case of capsule flow the Reynolds number to be applied to the fluid flowing in the annulus is not based on uav but on u'_{av}. In order to be able to readily calculate the value of this effective Reynolds number it is convenient to have a relationship between uav and u'_{av}.

For laminar flow the average velocity is one-half the maximum velocity and we may write

$$u'_{av} = 0.5u'_{max.} \quad (13)$$

And further, the capsule velocity in the form

$$u_c = u'_{max}\left(1-k^2\right) \quad (14)$$

Substituting Equation (13) in Equation (14) we have

$$u_c = 2u'_{av}(1-k^2) \quad (15)$$

and hence by combining Equation (15) with Equation (12) we have

$$u'_{av} = \frac{0.5u_{av}}{\pi k^2 - 4k^3 + \frac{3\pi}{16}k^4 + \frac{2k^2}{3}\left(1-k^2\right)} \quad (16)$$

RESULTS AND DISCUSSION

Results were obtained for the transport by water of single spherical capsules with density equal to that of the water in a 1.25 inch plastic pipe, capsule/pipe diameter ratio, k=d/D, equal to 0.39, 0.69 and 0.89. The average flow velocity varied from 0.11 to 0.23 ft/s corresponding to Re=1000 to 2000 (laminar flow).[Ellis and Bolt]

Figures (1,2 and 3) represent the relationship between the average flow velocity and capsule velocity they show a linear increase of capsule velocity as the average flow velocity is increased. This sensitivity of capsule velocity to average flow velocity appears to be limited to the boundary layer of the capsule. All capsule velocity are greater than the average flow velocities, it was pointed out that this was to be expected from consideration of the flow velocity profile, the velocity being zero at the pipe wall and a maximum on the pipe axis. These theoretical results were compared with experimental results that obtained by Ellis and Bolt [5], these comparisons show that the theoretical results have the same behavior of the experimental results. Also from these figures, one can observe the effect of diameter ratio on the capsule velocity in which the reduction of the diameter ratio of the spheres leads to a general increase in capsule velocity; the instability of the spheres is increased by decrease of diameter ratio, and the sensitivity of the capsule velocity to the disturbances in the capsule-free stream at the critical Reynolds numbers is also greatly increased.

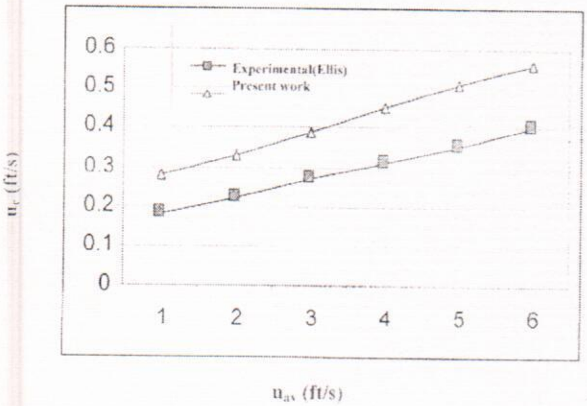

Figure (1) Variation of capsule velocity with average flow velocity for diameter ratio (k)=0.39

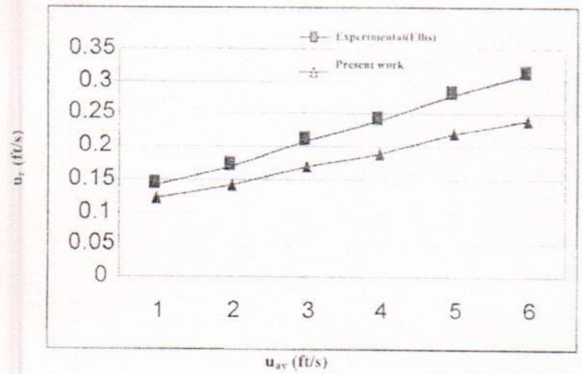

Figure (2) Variation of capsule velocity with average flow velocity for diameter ratio (k)=0.69

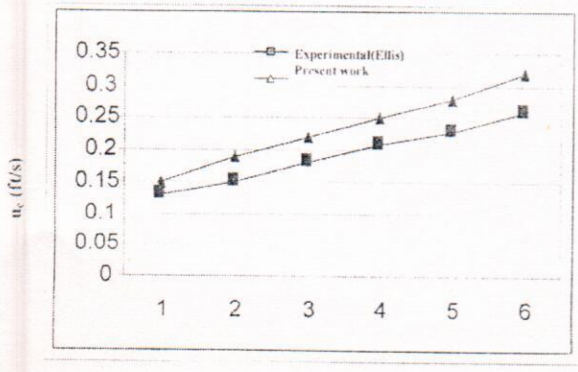

Figure (3) Variation of capsule velocity with average flow velocity for diameter ratio (k)=0.89

CONCLUSIONS

The following conclusions were obtained for the flow of single spherical capsules having the same density as the carrier fluid:

1. A reduction of diameter typically results in an increase of velocity of spherical capsule.
2. As the flow velocity is increased, the capsule velocity is also increased.
3. A good agreement in the behavior between the theoretical results (present work) and the experimental results obtained by Ellis and Bolt.

NOMENCLATURE

D	outside capsule diameter (in)
D	inside pipe diameter (in)
K	diameter ratio (d/D)
$\left(\frac{\Delta p}{L}\right)_c$	Capsule pressure gradient (N/m^3)
$Q_{annulus}$	Volumetric flow rate in the annulus (m^3/s)
$Q_{capsule}$	capsule volumetric flow rate (m^3/s)
μ	dynamic viscosity (N.S/m^2)
Re	pipe Reynolds number ($\rho u_{av} D/\mu$)
r_c	outside capsule radius (in)
ρ	fluid density (kg/m^3)
r_o	inside pipe radius (in)
u_{av}	average flow velocity (ft/s)
u'_{av}	average flow velocity by the presence of capsule (ft/s)
u_c	capsule velocity (ft/s).

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